Chapter 6. *pn*-junction diode: I-V characteristics

- **Topics:** steady state response of the *pn* junction diode under applied d.c. voltage.

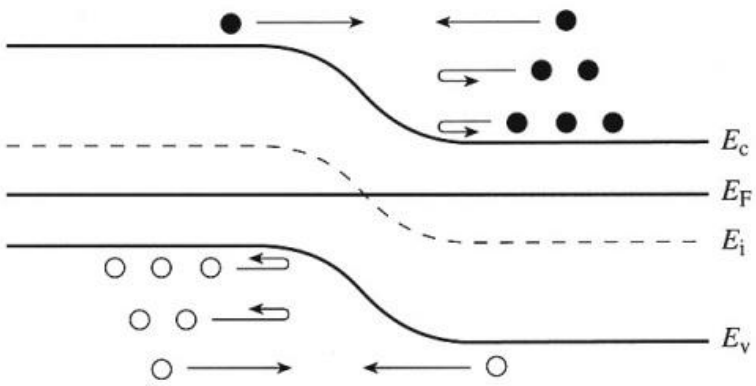
pn Junction under bias (qualitative discussion)

Ideal diode equation

Deviations from the ideal diode

Charge-control approach

Carrier flow in equilibrium



(a) Equilibrium ($V_A = 0$)

Electron diffusion current is precisely balanced by electron drift current.

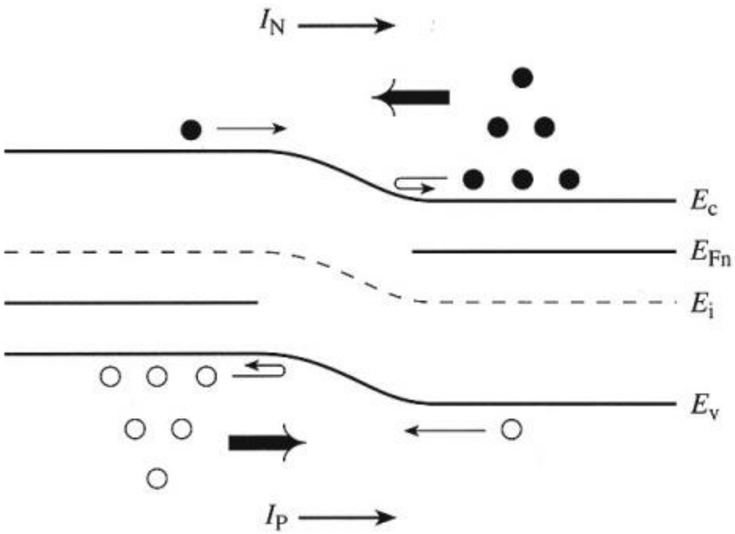
$$I_{n|drift} = I_{n|diffution}$$

hole diffusion current is also balanced by hole drift current.

$$I_{p|drift} = I_{p|diffution}$$

Thus, no net current across the junction.

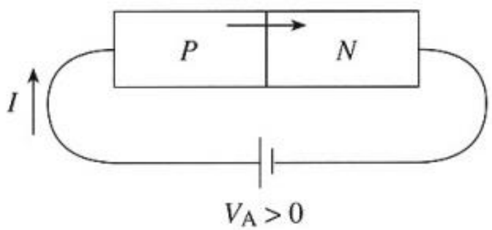
Carrier flow under forward bias



$$I_{n|drift} < I_{n|diffution}$$

$$I_{p|drift} < I_{p|diffution}$$

from p-to n-side

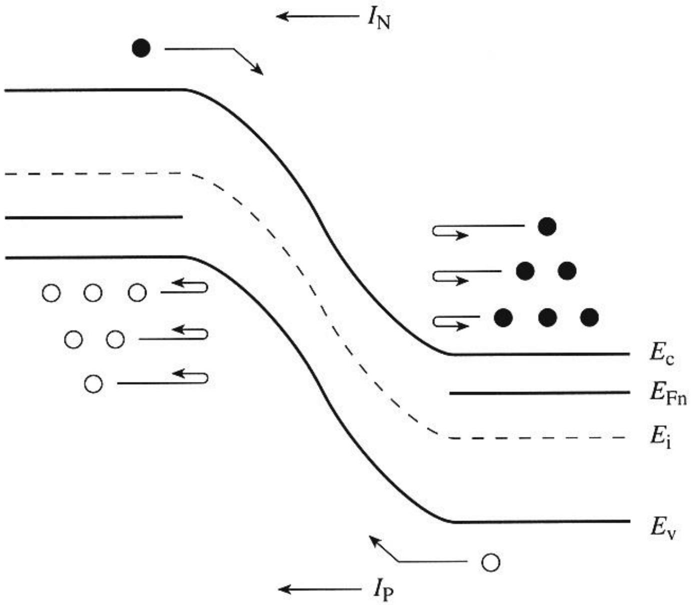


(b) Forward bias ($V_A > 0$)

As the potential hill linearly decreases with the forward bias, the number of majority carriers which have sufficient energy to surmount the potential barrier exponentially goes up with V_A.

It is expected that forward current (i.e., majority carrier diffusion current) exponentially increases with V_A.

Carrier flow under reverse bias



(c) Reverse bias ($V_A < 0$)

The majority carrier diffusion across the junction is negligible.

The minority carrier drift is still allowed to flow the reverse current (i.e., minority carrier drift current) across the junction (from n- to p-side).

The reverse current is expected to be extremely small in magnitude, due to the low concentration of the minority carriers.

As V_A negatively increases, the reverse current is also expected to saturate, once the majority carrier diffusion currents are reduce to a negligible level at a small bias.

Ch 6-1 The ideal diode equation

Net current = $I_{diff} - I_{drift}$

At equilibrium ($V_A = 0$), net current = 0

set $$\left|I_{diff}\right|_{V_A=0} = \left|I_{drift}\right|_{V_A=0} = I_0$$

$|I_{drift}|$ saturates and does not change with V_A (Why?)

Because the drift current is limited NOT by HOW FAST carriers are swept across the depletion layer, but rather HOW OFTEN. → think a waterfall !

$|I_{diff}|$ varies exponentially with V_A (Why?)

Because the number of carriers which have sufficient energy to surmount the potential barrier exponentially goes up with V_A.

Ch 6-1 The ideal diode equation

$|I_{diff}| = I_0 \exp(V_A/V_{ref})$ where I_0 and V_{ref} are constants.

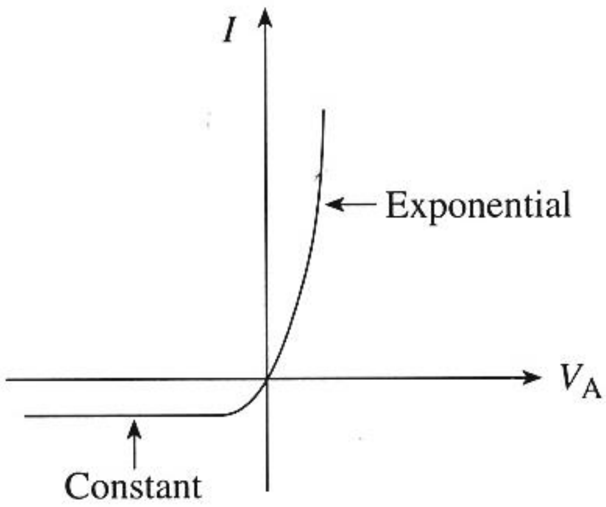
At any applied voltage, V_A,

$$I = I_0 e^{V_A/V_{ref}} - I_{drift}$$

since $I_{drift} = I_0$ at any voltage.

$$= I_0 e^{V_A/V_{ref}} - I_0$$

$$= I_0 \left(e^{V_A/V_{ref}} - 1\right)$$

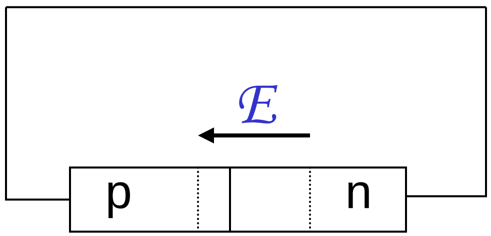


$$I = I_0 \left(e^{V_A/V_{ref}} - 1\right)$$

Predicted equation for ideal diodes

pn junction under various bias conditions

$V_A = 0$



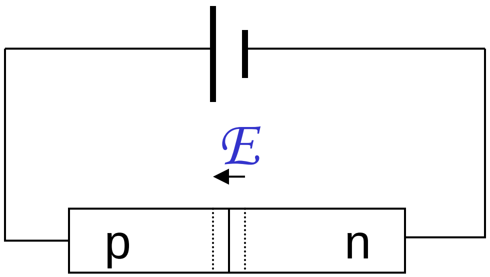
Majority hole diffusion current

Minority hole drift current

Majority electron diffusion current

Minority electron drift current

$V_A > 0$



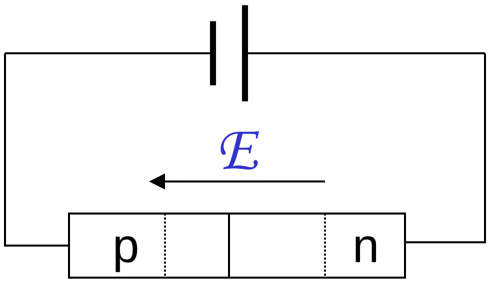
Majority hole diffusion current

Minority hole drift current

Majority electron diffusion current

Minority electron drift current

$V_A < 0$



Majority hole diffusion current

Minority hole drift current

Majority electron diffusion current

Minority electron drift current

Ideal diode equation: quantitative solution

- Assumptions which must hold
  - The diode is being operated under steady state conditions
  - A non-degenerately doped step junction models the doping profile
  - The diode is one-dimensional
  - Low-level injection prevails in the quasi-neutral regions
  - There are no processes other than drift, diffusion, and thermal recombination-generation taking place inside the diode, specifically, $G_L = 0$

Ideal diode equation: quantitative solution

We want to obtain a current equation of diode against V_A.

Therefore the total current can be obtained from the total current density (J).

$$I = AJ$$

Note that the total current density (J) is constant throughout the diode under the steady state, but the J_n and J_p vary with position.

$$J = J_n(x) + J_p(x)$$

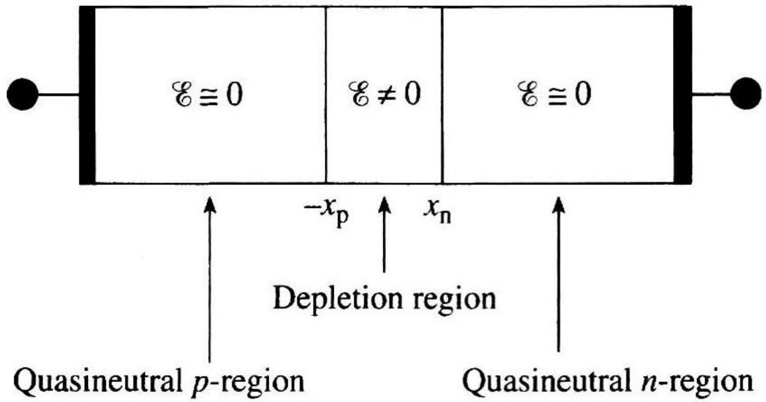
The $J_n(x)$ and $J_p(x)$ should be expressed as a function of x by using the following equations,

$$J_n = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx} \qquad J_p = q\mu_p p\mathcal{E} + qD_p \frac{dp}{dx}$$

The n and p can be evaluated by using the continuity equation.

Ideal diode equation: quantitative solution

Quasi-neutral region consideration



Let's consider the $J_n(x)$ and $J_p(x)$ in the quasi-neutral regions, because the continuity equation can be simplified to the minority carrier diffusion equation in this region (note that $\mathscr{E} \approx 0$ and the low level injection assumption).

Minority carrier diffusionequaiton

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

for electrons in *p*-type

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

for holes in *n*-type

Ideal diode equation: quantitative solution

Quasi-neutral region consideration

Under the assumption of the steady state with $G_L = 0$,

Already we know the general solution

$$0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} \qquad x \le -x_p$$

$$0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} \qquad x \ge x_n$$

$$\Delta n_p(x) = Ae^{-x/L_n} + Be^{x/L_n} \qquad \Delta p_n(x) = Ae^{-x/L_p} + Be^{x/L_p}$$

Since $\mathcal{E} \approx 0$ and $dn_0/dx = dp_0/dx = 0$ in the quasi-neutral region, (note that $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$)

$$J_n = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx} \longrightarrow J_n = qD_n \frac{d\Delta n_p}{dx} \qquad x \le -x_p$$

$$J_p = q\mu_p p\mathcal{E} + qD_p \frac{dp}{dx} \longrightarrow J_p = qD_p \frac{d\Delta p_n}{dx} \qquad x \ge x_n$$

Ideal diode equation: quantitative solution

Depletion region consideration

In the depletion region, $\mathcal{E} \neq 0$ so, the continuity equation must be used under our assumptions (steady state and only thermal R-G process).

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + \left.\frac{\partial n}{\partial t}\right|_{\substack{thermal \\ R-G}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{others \\ (light\,etc.)}} \longrightarrow 0 = \frac{1}{q}\frac{\partial J_n}{\partial x} + \left.\frac{\partial n}{\partial t}\right|_{\substack{thermal \\ R-G}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + \left.\frac{\partial p}{\partial t}\right|_{\substack{thermal \\ R-G}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{others \\ (light\,etc.)}} \longrightarrow 0 = -\frac{1}{q}\frac{\partial J_p}{\partial x} + \left.\frac{\partial p}{\partial t}\right|_{\substack{thermal \\ R-G}}$$

Additionally, we can assume that thermal R-G process is negligible throughout the depletion region.

Thus, $0 = \frac{\partial J_n}{\partial x}$ and $0 = \frac{\partial J_p}{\partial x}$ at $-x_p \leq x \leq x_n$

Ideal diode equation: quantitative solution

Depletion region consideration

$$0 = \frac{\partial J_n}{\partial x} \quad \text{and} \quad 0 = \frac{\partial J_p}{\partial x} \quad \text{at} \quad -x_p \le x \le x_n$$

This reveals the constancy of the carrier currents throughout the depletion region (including the edges).

$$J_n(-x_p \le x \le x_n) = J_n(-x_p) = J_n(x_n)$$

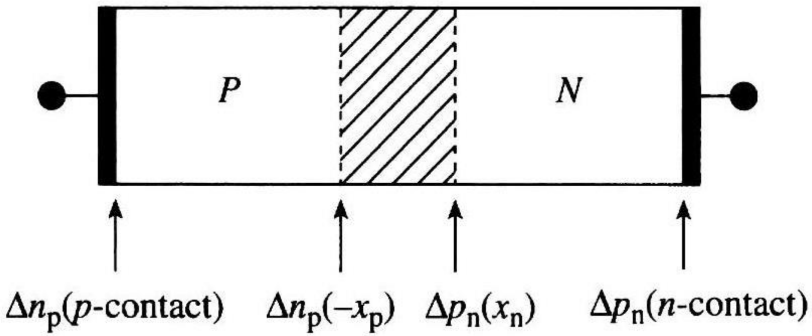
$$J_p(-x_p \le x \le x_n) = J_p(-x_p) = J_p(x_n)$$

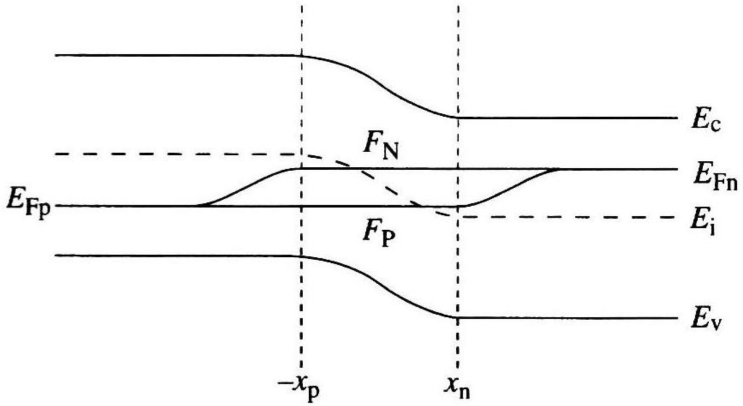
Summing two equations,

$$\boxed{J = J_n(-x_p) + J_p(x_n)}$$

Ideal diode equation: quantitative solution

Boundary conditions





[Band diagram inside a forward-biased diode]

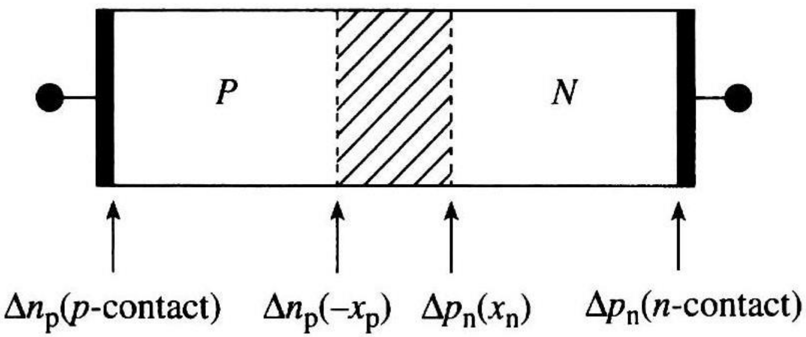
If the ohmic contacts are far enough from the edges of the depletion region, the boundary conditions at the ohmic contacts will be

$$\Delta n_p(x \to -\infty) = 0$$

$$\Delta p_n(x \to +\infty) = 0$$

Ideal diode equation: quantitative solution

Boundary conditions



To establish the boundary conditions at the edges of the depletion region, consider the definition of the quasi-Fermi levels.

$$F_P \equiv E_i - kT\ln\left(\frac{p}{n_i}\right)$$

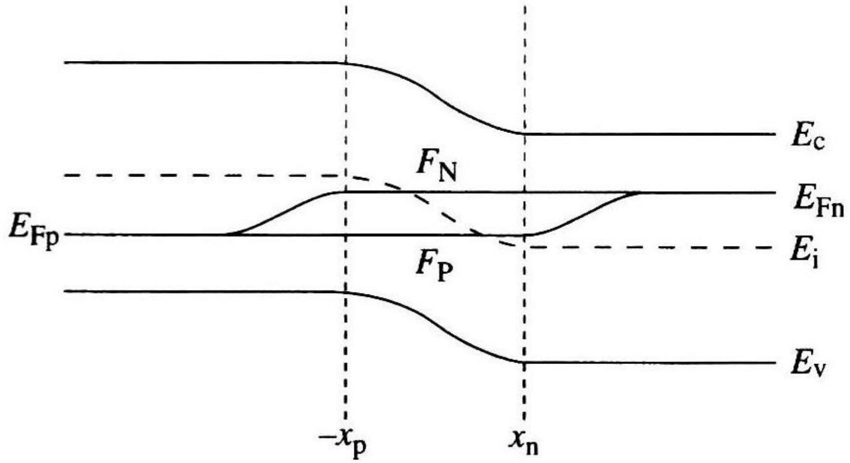
$$F_N \equiv E_i + kT\ln\left(\frac{n}{n_i}\right)$$

$$n = n_i e^{\left(\frac{F_n - E_i}{kT}\right)} \qquad p = n_i e^{\left(\frac{E_i - F_p}{kT}\right)}$$

Thus, $$np = n_i^2 e^{\left(\frac{F_n - F_p}{kT}\right)}$$

Ideal diode equation: quantitative solution

Boundary conditions



$$np = n_i^2 e^{\left(\frac{F_n - F_p}{kT}\right)}$$

$$E_{Fn} - E_{Fp} = qV_A$$

[F_p and F_n variation inside a forward-biased diode]

Assuming $F_n - F_p = qV_A$, $F_n = E_{Fn}$ and $F_p = E_{Fp}$ throughout the depletion region,

$$F_n - F_p \le E_{Fn} - E_{Fp} = qV_A$$

Therefore, $$np = n_i^2 e^{qV_A/kT} \qquad -x_p \le x \le x_n$$

This equation is referred to as the "**law of the junction**".

Ideal diode equation: quantitative solution

Boundary conditions

$$\boxed{np = n_i^2 e^{qV_A/kT}} \qquad -x_p \le x \le x_n$$

At the p-edge of the depletion region,

$$n(-x_p)p(-x_p) = n(-x_p)N_A = n_i^2 e^{qV_A/kT} \qquad \text{or} \qquad n(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

Then, $\Delta n_p(-x_p) = n_p(-x_p) - n_{p_0}(-x_p) = \dfrac{n_i^2}{N_A} e^{qV_A/kT} - \dfrac{n_i^2}{N_A}$

$$\boxed{\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)}$$

Similarly at the n-edge, $\boxed{\Delta p_n(x_n) = \dfrac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)}$

Plan for the quantitative solution

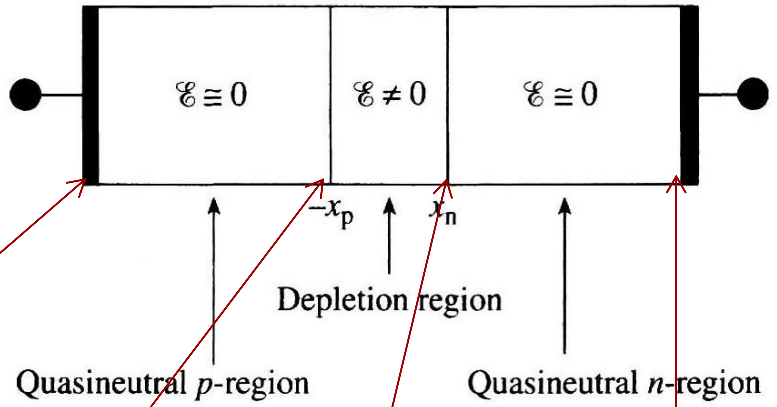
1) Solve the minority carrier diffusion equations employing boundary conditions

$$0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} \qquad x \le -x_p$$

$$0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} \qquad x \ge x_n$$

$$\Delta n_p(x) = Ae^{-x/L_n} + Be^{x/L_n}$$

$$\Delta p_n(x) = Ae^{-x/L_p} + Be^{x/L_p}$$



$$\Delta p_n(x \to +\infty) = 0$$

$$\Delta n_p(x \to -\infty) = 0$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Plan for the quantitative solution

2) Compute the minority carrier current densities in the quasi-neutral regions using

$$\begin{cases} J_n = qD_n \dfrac{d\Delta n_p}{dx} & x \le -x_p \\ J_p = qD_p \dfrac{d\Delta p_n}{dx} & x \ge x_n \end{cases}$$

3) Evaluate the quasi-neutral region solutions for $J_n(x)$ and $J_p(x)$ at the edges of the depletion region and then sum the two edge current densities.

$$\boxed{J = J_n(-x_p) + J_p(x_n)}$$

4) Finally, multiply the result by the cross-sectional area of the diode.

$$I = AJ$$

Announcements

- Next lecture: p. 247 ~ 259