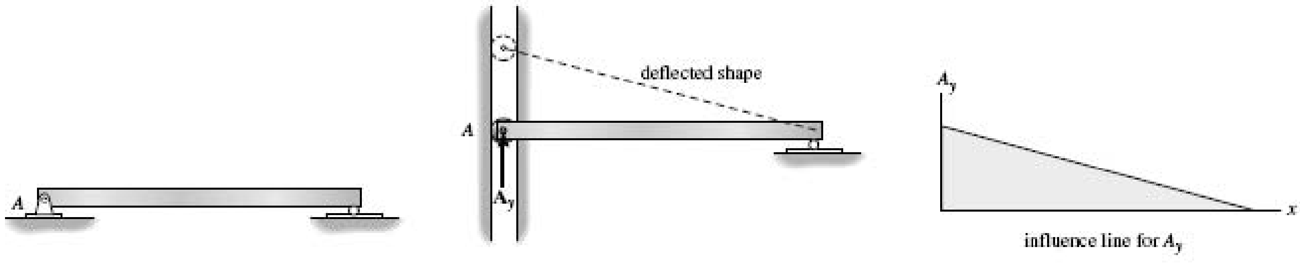
3. Qualitative influence line

In 1886, Heinrich Müller- Breslau developed a technique for rapidly constructing the shape of influence line.

- The Müller-Breslau Principle states that the influence line for a function is to the same scale as the deflected shape of the beam when the beam is acted upon by the function
- If the shape of the influence line for the vertical reaction at *A* is to be determined, the pin is first replaced by a roller guide
- When the +ve force $\mathbf{A}_y$ is applied at *A*, the beam deflects to the dashed position which represents the

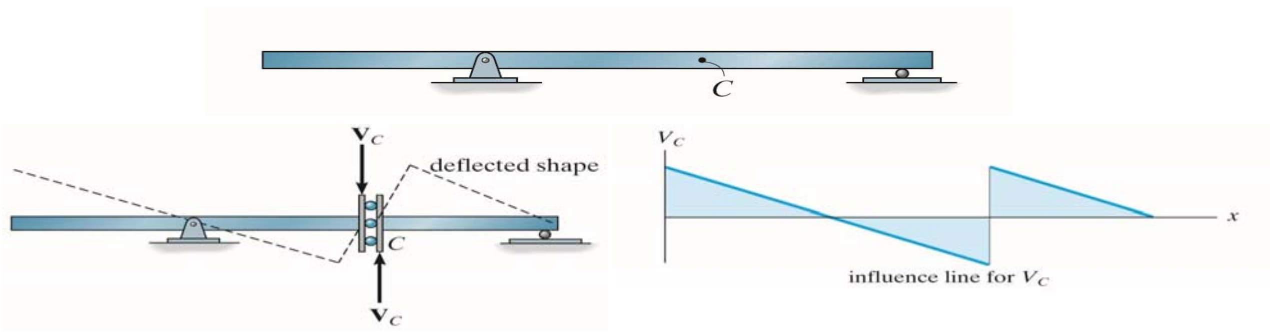


(a) (b) (c)

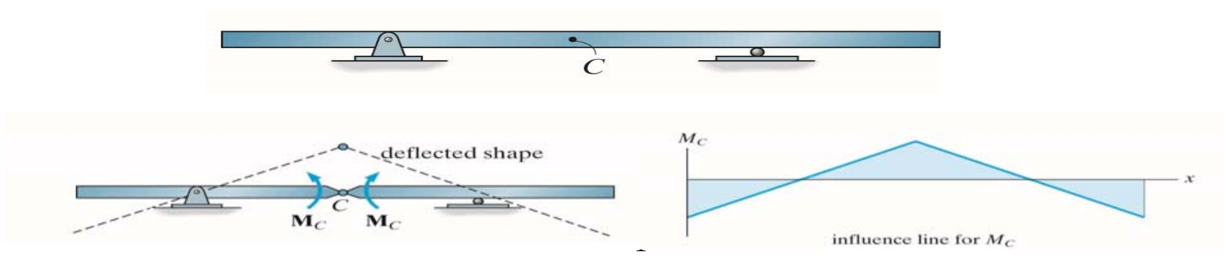
Fig. 6–12

general shape of the influence lineA

- If the shape of the influence line for shear at *C* is to be determined, the connection at *C* may be symbolized by a roller guide
- Applying a +ve shear force $\mathbf{V}_c$ to the beam at *C* & allowing the beam to deflect to the dashed position

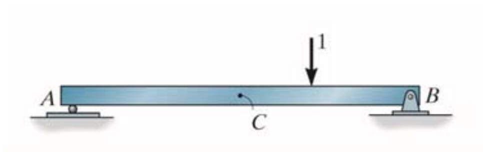


- If the shape of influence line for the moment at *C* is to be determined, an internal hinge or pin is placed at *C*
- Applying +ve moment $\mathbf{M}_c$ to the beam, the beam deflects to the dashed line



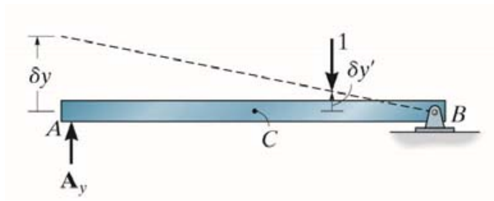
Proof of the Müller-Breslau principle using the principle of virtual work

- The proof of the Müller-Breslau Principal can be established using the principle of virtual work
- Work = a linear displacement x force in the direction of displacement
- Or work = rotational displacement x moment if the direction of the displacement
- If a rigid body is in equilibrium, the sum of all the forces & moments on it must be equal to zero
- If the body is given an imaginary or virtual displacement, work done by all these forces & couple moments must also be equal to zero



(1) Reaction Ay

- If the beam shown is given a virtual displacement δy at the support A, then only $\mathbf{A}_y$ & unit load do virtual work
- A_y does +ve work = $A_y\ \delta y$
- The unit load does –ve work = -1 δy'



- Since the beam is in equilibrium, the virtual work sums to zero

$$A_y \delta y - 1\delta y' = 0$$

$$\text{If } \delta y = 1, \text{ then} \Rightarrow A_y = \delta y'$$

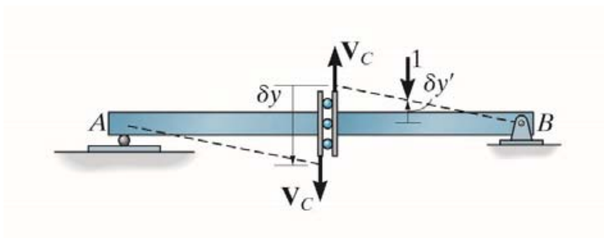
- The value of A_y represents the ordinate of the influence line at the position of the unit load

(2) Shear at C

- If the beam is sectioned at C, the beam undergoes a virtual displacement δy then only the internal shear at C and the unit load do work
- The virtual work eqn is:

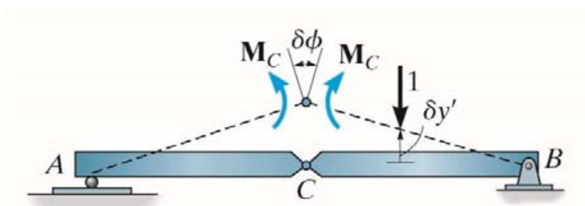
$$V_c \delta y - 1\delta y' = 0$$

$$\text{If } \delta y = 1, \text{ then} \Rightarrow V_c = \delta y'$$



- The shape of the influence line for shear at C has been established

(3) Moment at C



- If a virtual rotation $\delta\phi$ is introduced at the pin, virtual work will be done only by the internal moment & unit load

$$M_c\delta\phi - 1\delta y' = 0$$

If $\delta\phi = 1$, then $\Rightarrow M_c = \delta y'$

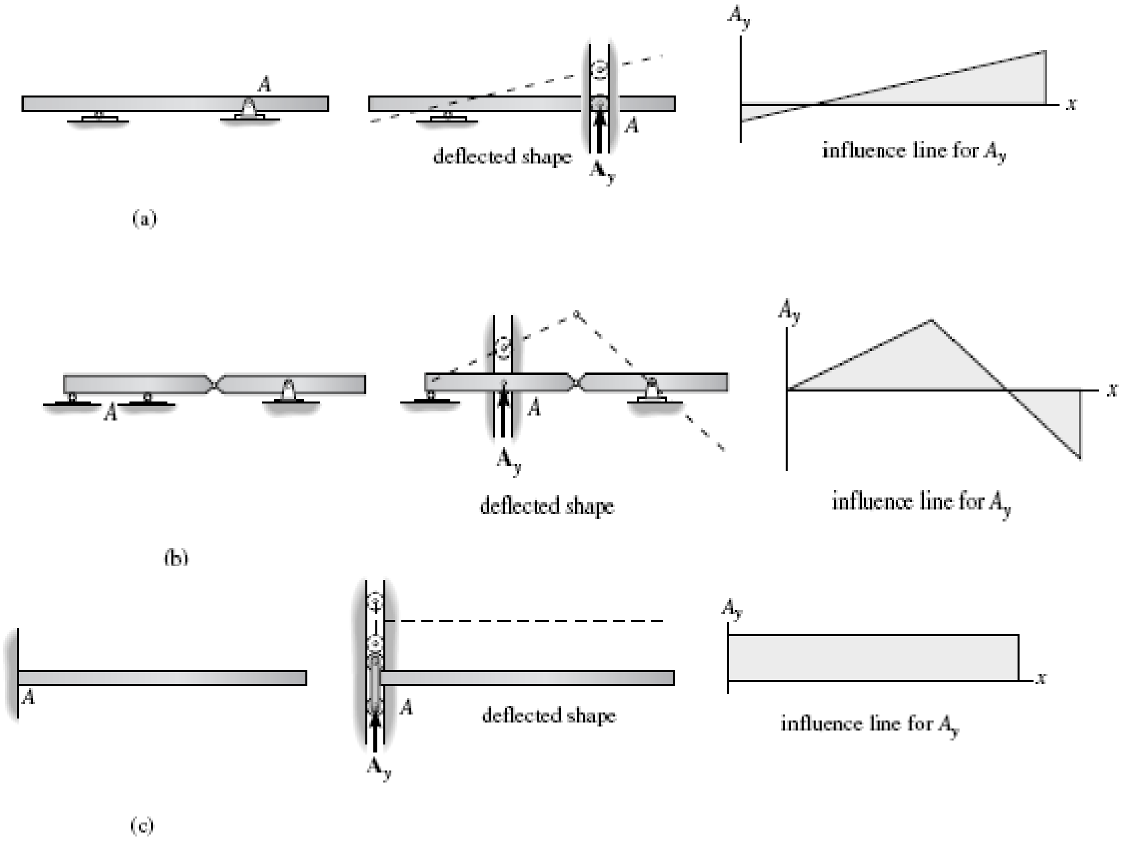
Example 6.9

For each beam in Fig.6-16a through 6-16c, sketch the influence line for the vertical reaction at A.

[solution]

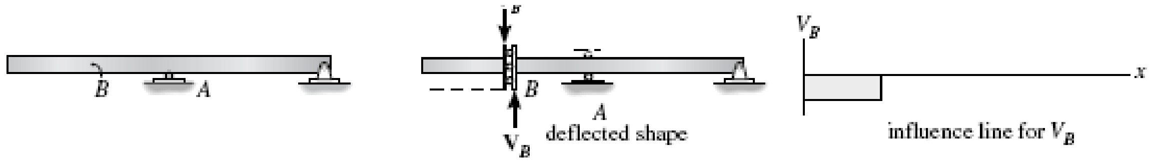
The support is replaced by a roller guide at A and the force A_y is applied. Again, a roller guide is placed at A and the force A_y is applied.

A double-roller guide must be used at A in this case, since this type of support will then transmit both a moment M_A at the fixed support and force A_x, but will not transmit A_y



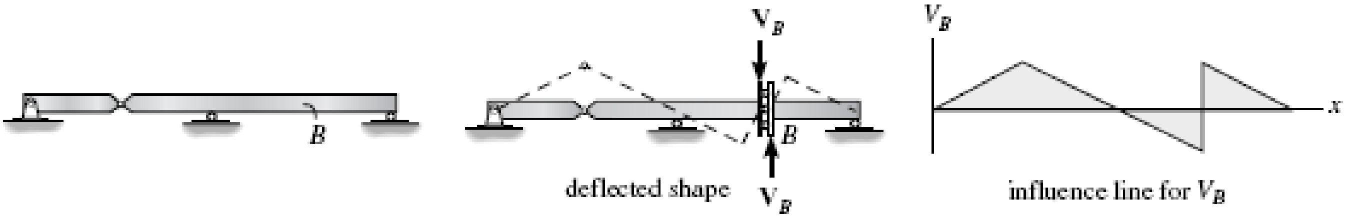
Example 6.10
For each beam in Fig.6-17a through 6-17c, sketch the influence line for the shear at B.

[solution]
The roller guide is introduced at B and the positive shear V_B is applied. Notice that the right segment of the beam will not deflect since the roller at A actually constrains the beam from moving vertically, either up or down. [See support (2) in Table 2-1.]



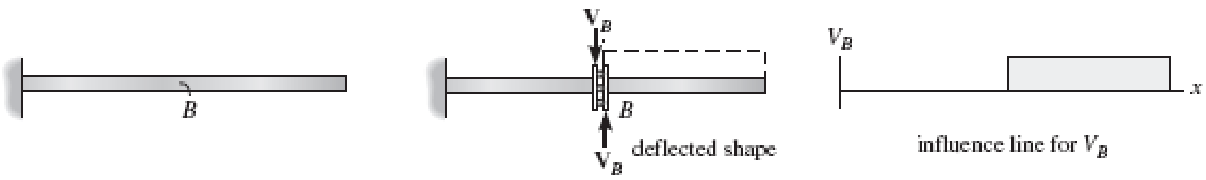
(a)

Placing the roller guide at B and applying the positive shear at B yields the deflected shape and corresponding influence line.



(b)

Again, the roller guide is placed at B, the positive shear is applied, and the deflected shape and corresponding influence line are shown. Note that the left segment of the beam does not deflect, due to the fixed support.



(c)