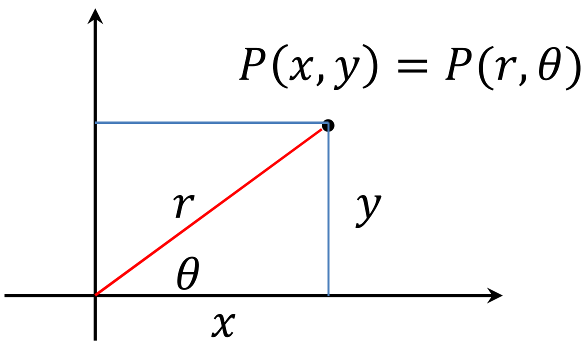
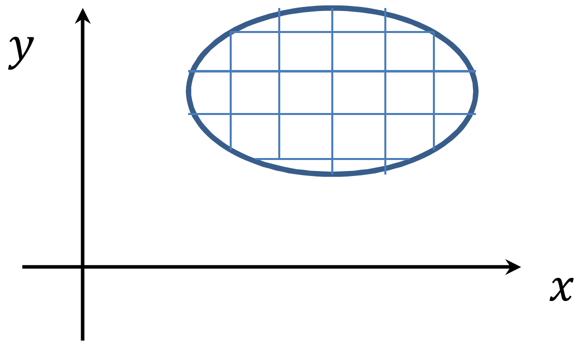
15.3 극좌표에서의 이중적분

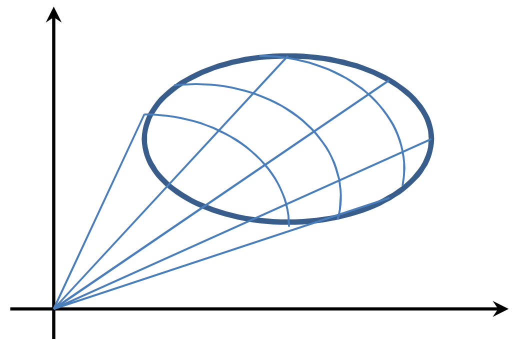
직교좌표와 극좌표



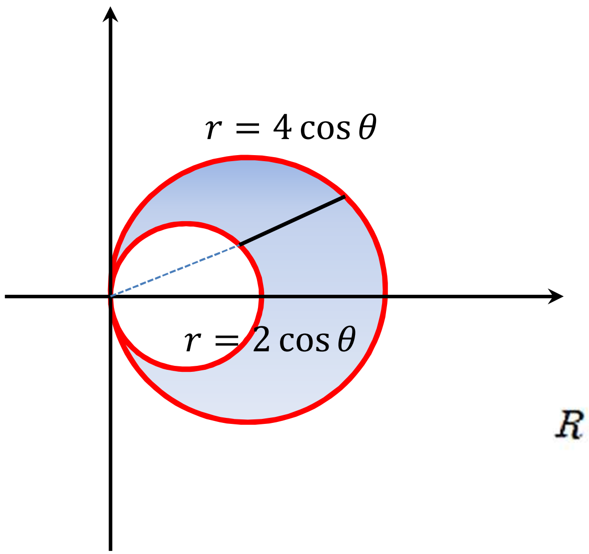
$$x = r\cos\theta$$

$$y = r\sin\theta$$



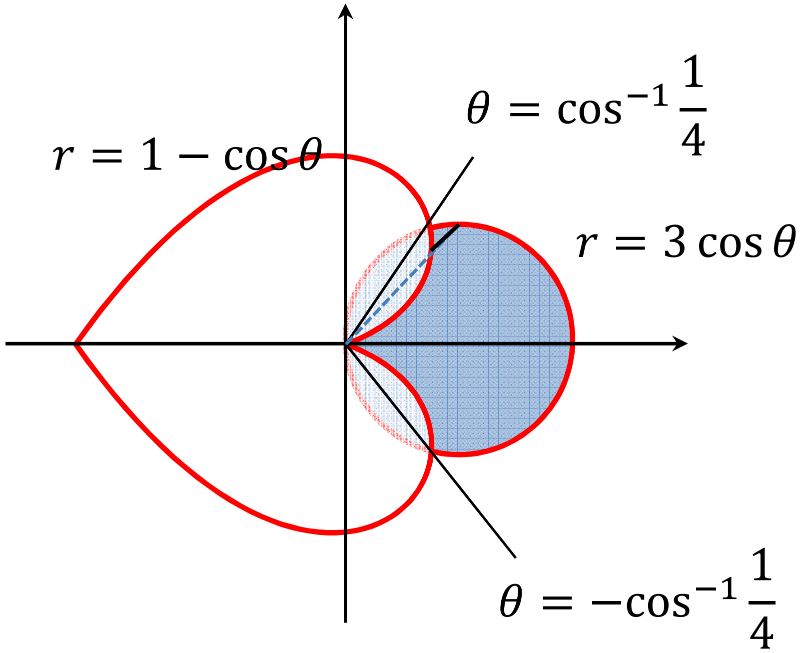


Example $r = 2\cos\theta$ 와 $r = 4\cos\theta$ 사이의 영역 R 을 극좌표를 써서 나타내어라.



$$R = \left\{ (r,\theta) \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ 2\cos\theta \le r \le 4\cos\theta \right\}$$

Example $r = 3\cos\theta$ 의 내부에 있고, 심장형 $r = 1 - \cos\theta$ 의 외부에 있는 영역 R

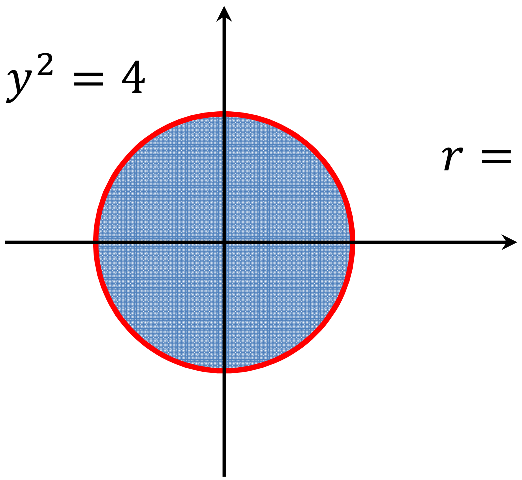


$$R = \left\{(r,\theta) \middle| 1 - \cos\theta \le r \le 3\cos\theta,\ -\cos^{-1}\frac{1}{4} \le \theta \le \cos^{-1}\frac{1}{4}\right\}$$

Example

$$R = \left\{(x, y) \middle| -\sqrt{4 - y^2} \leq x \leq \sqrt{4 - y^2}, -2 \leq y \leq 2\right\}$$

$$x^2 + y^2 = 4$$

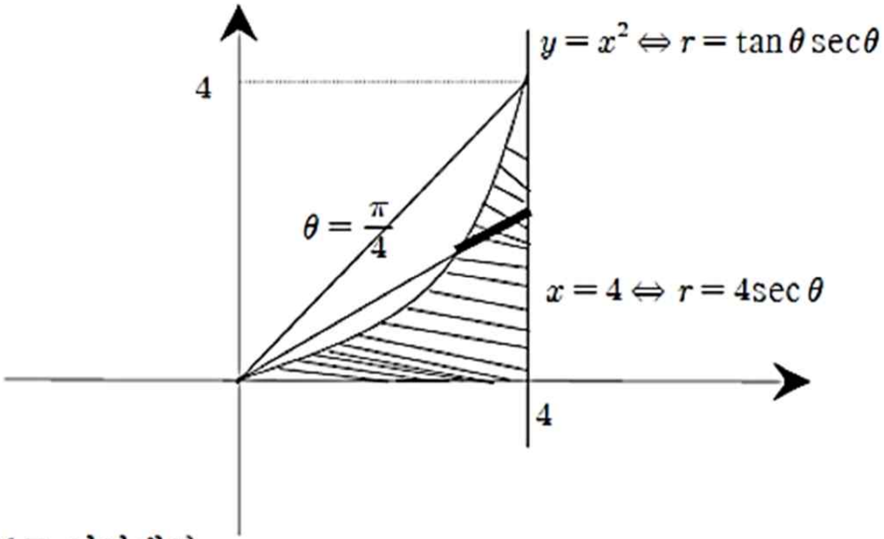


$$r = 2$$

$$R = \{(r, \theta) | 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi\}$$

Example $R = \{(r,\theta) | \sqrt{y} \le x \le 2, 0 \le y \le 4\}$

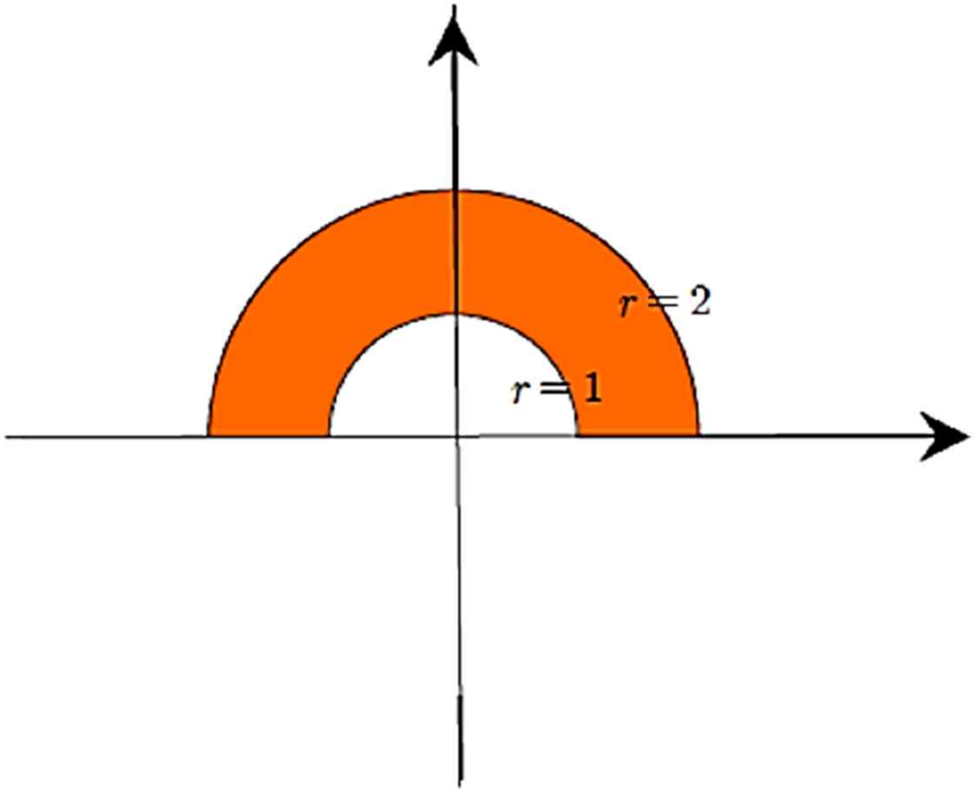
$R = \{(r,\theta) \mid \sqrt{y} \le x \le 2,\ 0 \le y \le 4\}$ 의 그림은



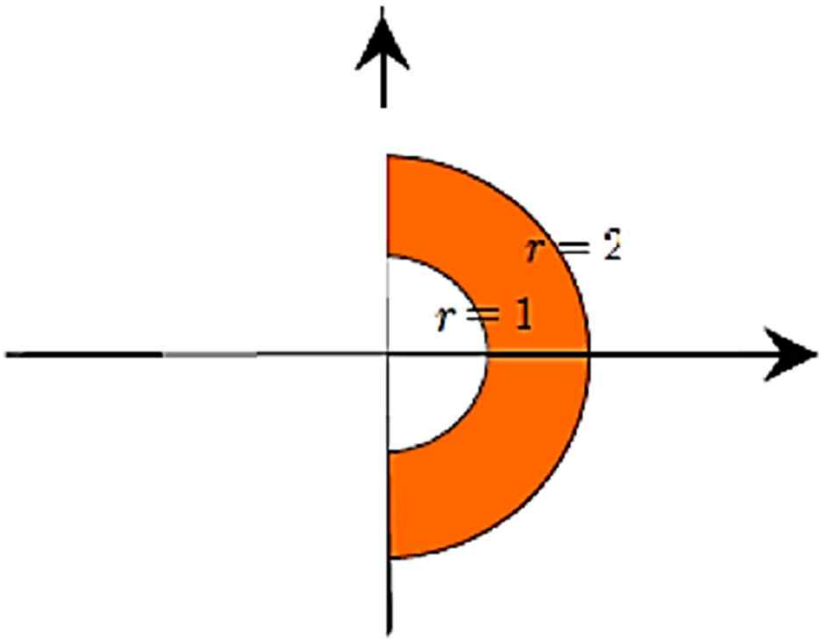
이고 극좌표로 나타내면

$$R = \left\{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{4},\ \tan\theta\sec\theta \le r \le 4\sec\theta\right\}$$

$R = \{(r,\theta) \mid 0 \le \theta \le \pi,\ 1 \le r \le 2\}$ 의 그림은

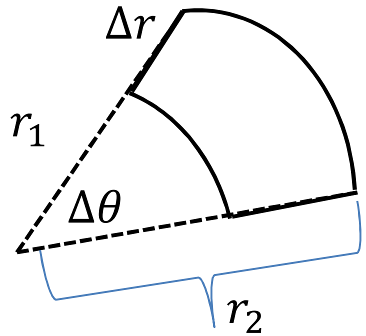


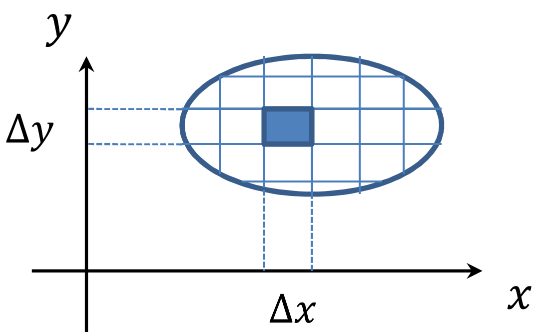
$R=\left\{(r,\theta)\mid -\frac{\pi}{4}\leq\theta\leq\frac{\pi}{4},\ 1\leq r\leq 2\right\}$ 의 그림은



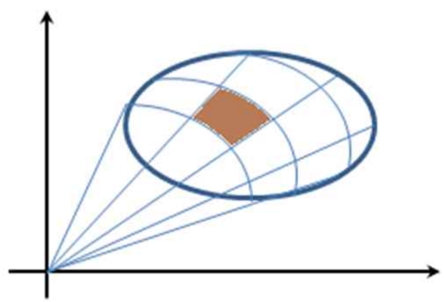
극좌표에서의 이중적분

$$\iint_{R_{xy}} f(x,y)dA = \sum f(x^*, y^*)\Delta x \Delta y$$

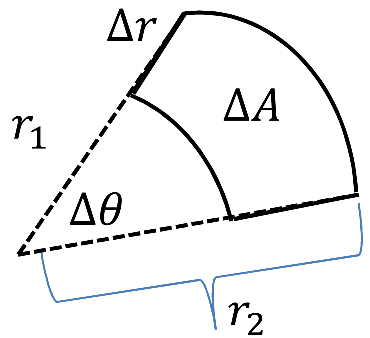


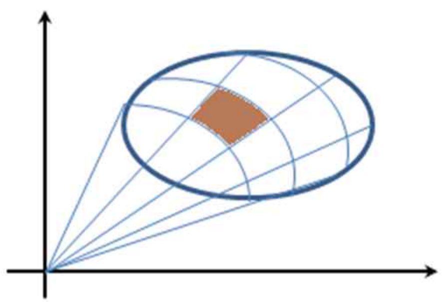


R_{xy}



R_{θ}





$$\Delta A = \frac{1}{2} r_2^2 \Delta\theta - \frac{1}{2} r_1^2 \Delta\theta$$

$$= \frac{1}{2}(r_2^2 - r_1^2)\Delta\theta$$

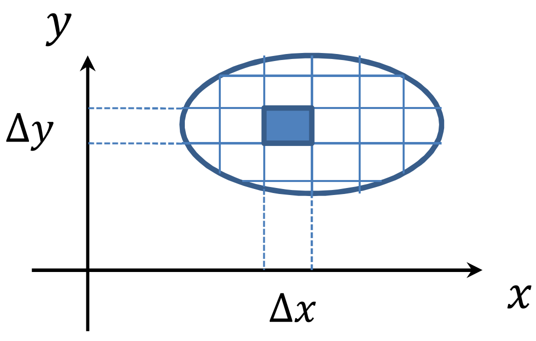
$$= \frac{1}{2}(r_2 + r_1)(r_2 - r_1)\Delta\theta$$

$$= \frac{1}{2}(r_2 + r_1)\Delta r \Delta\theta$$

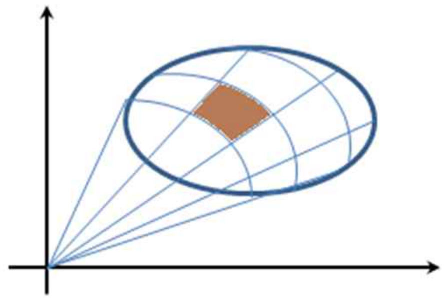
$$= r^* \Delta r \Delta\theta$$

$$\iint_{R_{xy}} f(x,y)\,dA = \sum f(x^*,y^*)\Delta x \Delta y \quad = \sum f(r^*\cos\theta, r^*\sin\theta)\, r^*\Delta r \Delta\theta$$

$$= \iint_{R_\theta} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$



R_{xy}

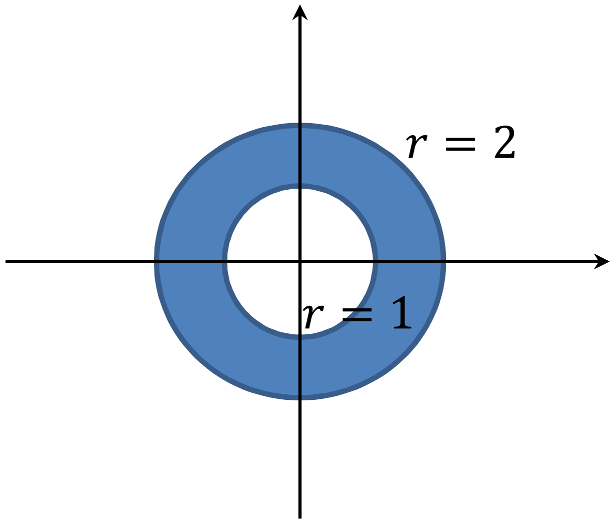


R_θ

Example 영역 R_{xy} 이 원 $x^2 + y^2 = 1$ 과 $x^2 + y^2 = 4$ 사이 일 때,
다음 이중적분을 계산하여라.

$$\iint_{R_{xy}} (3x + 4y^2)\,dA$$

$$\int_{R_{xy}} (3x + 4y^2)dA = \int_0^{2\pi}\int_1^2 (3r\cos\theta + 4r^2\sin^2\theta)r\,dr\,d\theta$$



$1 \le r \le 4$ $\qquad$ $0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_1^2 (3r^2\cos\theta + 4r^3\sin^2\theta)dr\,d\theta$$

$$= \int_0^{2\pi} \left[r^3\cos\theta + r^4\sin^2\theta\right]_1^2 d\theta$$

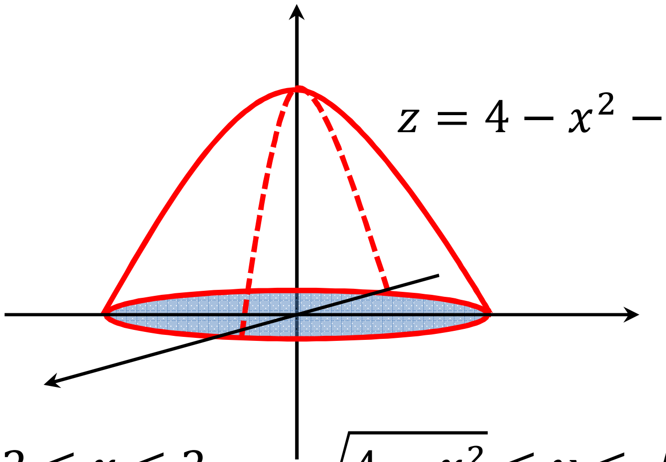
$$= \int_0^{2\pi} (7\cos\theta + 15\sin^2\theta)d\theta$$

$$= \int_0^{2\pi} (7\cos\theta + \frac{15}{2}(1 - \cos 2\theta)d\theta$$

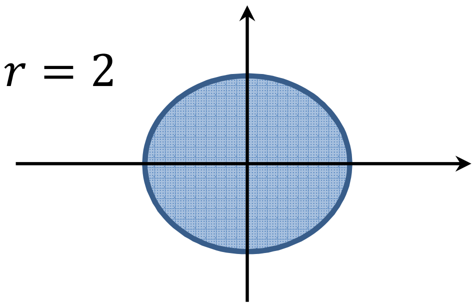
$$= \left[7\sin\theta + \frac{15\theta}{2} - \frac{15\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= 15\pi$$

Example 원포물면 $z = 4 - x^2 - y^2$ 과 xy-평면으로 둘러싸인 입체의 부피를 구하여라.



$-2 \le x \le 2 \quad -\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$



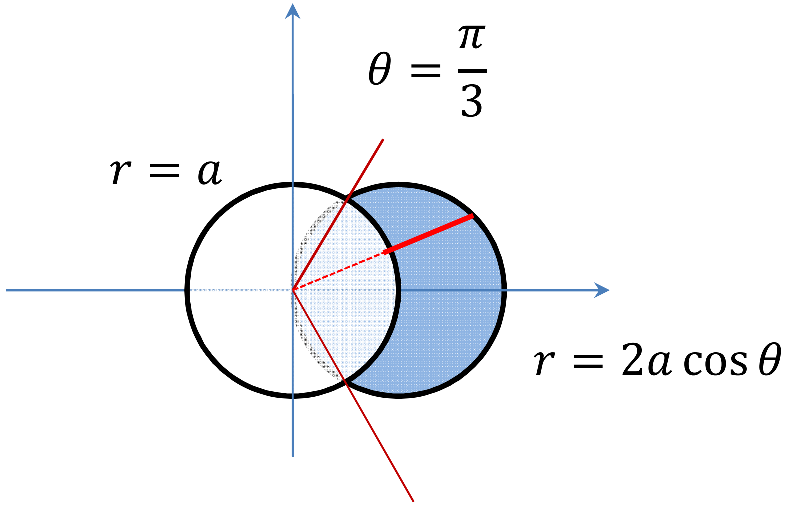
$0 \le r \le 2, \quad 0 \le \theta \le 2\pi$

$$
\begin{aligned}
\text{부피} &= \int_{-2}^{2}\int_{\sqrt{4-x^2}}^{-\sqrt{4-x^2}} (4 - x^2 - y^2)\,dydx \\
&= 4\int_0^{\frac{\pi}{2}}\int_0^2 (4 - r^2)\,rdrd\theta \\
&= 4\int_0^{\frac{\pi}{2}}\int_0^2 4d\theta \\
&= 8\pi
\end{aligned}
$$

Example 원 $r = 2a\cos\theta$ 의 내부와 원 $r = a$ 의 외부로 이루어진 부분의 면적을 이중적분을 이용하여 구하여라.

적분영역 R 의 넓이

$$A = \iint_R dA = \iint_{R_\theta} r dr d\theta$$



교점

$$a = 2a\cos\theta \Longrightarrow \theta = \pm\frac{\pi}{3}$$

$$a \le r \le 2a\cos\theta$$

$$-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$$

$$A = \iint_R dA = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_a^{2a\cos\theta} r\,dr\,d\theta$$

$$= 2\int_0^{\frac{\pi}{3}} \int_a^{2a\cos\theta} r\,dr\,d\theta$$

$$= 2\int_0^{\frac{\pi}{3}} \left[\frac{1}{2}r^2\right]_a^{2a\cos\theta} d\theta$$

$$= \int_0^{\frac{\pi}{3}} (4a^2\cos^2\theta - a^2)\,d\theta$$

$$= \int_0^{\frac{\pi}{3}} (a^2 + 2a^2\cos 2\theta)\,d\theta$$

$$= \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)a^2$$

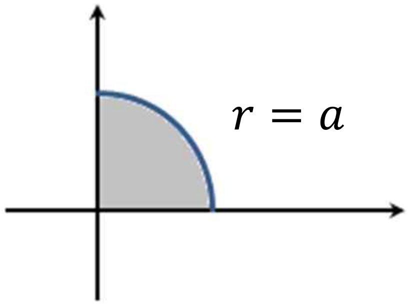
적분영역을 극좌표로!!

Example 다음 이중적분을 구하여라

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)} dydx$$

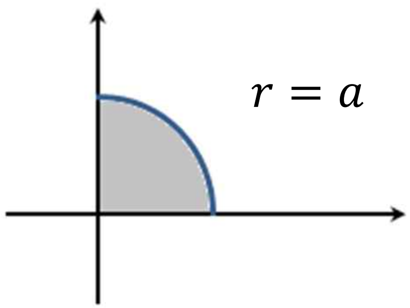
$0 \leq x \leq a$

$0 \leq y \leq \sqrt{a^2 - x^2}$



$0 \leq r \leq a$

$0 \leq \theta \leq \frac{\pi}{4}$



$r = a$

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)} dy dx$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2}\, r dr d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[-\frac{1}{2} e^{-r^2} \right]_0^a d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \left(1 - e^{-a^2}\right) d\theta$$

$$= \frac{\pi}{4} \left(1 - e^{-a^2}\right)$$

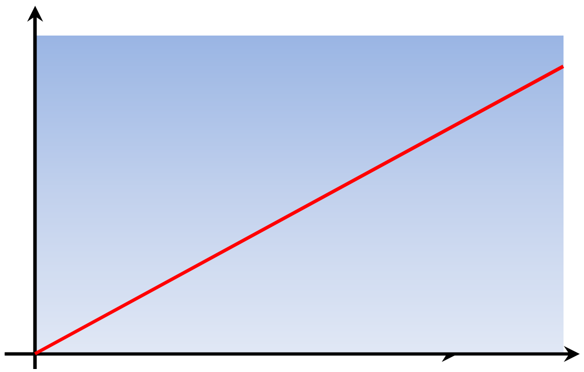
$0 \leq r \leq a$

$0 \leq \theta \leq \frac{\pi}{2}$

Example

$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$$

$0 \le x \le \infty$ $\quad$ $0 \le y \le \infty$



$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^2}\, rdrd\ \theta$$

$0 \le r < \infty$ $\quad$ $0 \le \theta \le \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-r^2}\, r dr d\theta$$

$$\int_0^{\infty} e^{-r^2}\, r dr = \lim_{b\to\infty}\int_0^{b} e^{-r^2}\, r dr$$

$$= \lim_{b\to\infty}\left[-\frac{1}{2}e^{-r^2}\right]_0^b$$

$$= \lim_{b\to\infty}\left[\frac{1}{2}-\frac{1}{2}e^{-b^2}\right] = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^2}\, r dr d\ \theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} d\theta$$

$$= \frac{\pi}{4}$$

$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dxdy$$

$$= \int_0^{\infty} e^{-y^2}\int_0^{\infty} e^{-x^2} dxdy$$

$$= \int_0^{\infty} e^{-x^2} dx \int_0^{\infty} e^{-y^2} dy$$

$$= \frac{\pi}{4}$$

$$\boxed{\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}}$$

Example

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dxdy$$

풀이. 적분영역을 나타내면

$$\begin{aligned} R &= \{(x,y) \mid -\infty \le x \le \infty, \quad -\infty \le y \le \infty\} \\ &= \{(r,\theta) \mid 0 \le \theta \le 2\pi, \quad 0 \le r \le \infty\} \end{aligned}$$

이므로 주어진 적분을 극좌표계에서 계산하면

$$\begin{aligned} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}dxdy &= \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r dr d\theta \\ &= \int_0^{2\pi} \lim_{r\to\infty}\left[-\frac{1}{2}e^{-r^2}\right]_0^r d\theta \\ &= \pi \end{aligned}$$

Example

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

(1) $\int_0^{\infty} e^{-4x^2} dx = \frac{\sqrt{\pi}}{4}$

(2) $\int_0^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

(1) 치환 $2x = t$로 치환하면

$$\int_0^{\infty} e^{-4x^2} dx = \frac{1}{2}\int_0^{\infty} e^{-t^2} dt = \frac{1}{2}\frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{4}$$

(2) 부분적분

$u = x,\ v' = xe^{-x^2}$ 라 하면

$$\begin{aligned} u &= x & v' &= xe^{-x^2} \\ u' &= 1 & v &= -\tfrac{1}{2}e^{-x^2} \end{aligned}$$

$$\int_0^{\infty} x^2 e^{-x^2} dx = \lim_{b\to\infty}\left[-\frac{1}{2}xe^{-x^2}\right]_0^b + \frac{1}{2}\int_0^{\infty} e^{-x^2} dx$$
$$= \frac{\sqrt{\pi}}{4}$$

(3) $\displaystyle\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx = \sqrt{\pi}$

(4) $\displaystyle\int_0^\infty \sqrt{x}e^{-x}dx = \frac{\sqrt{\pi}}{4}$

(3) $\sqrt{x}=t$로 치환하면 $\frac{1}{\sqrt{x}}dx = 2dt$이므로

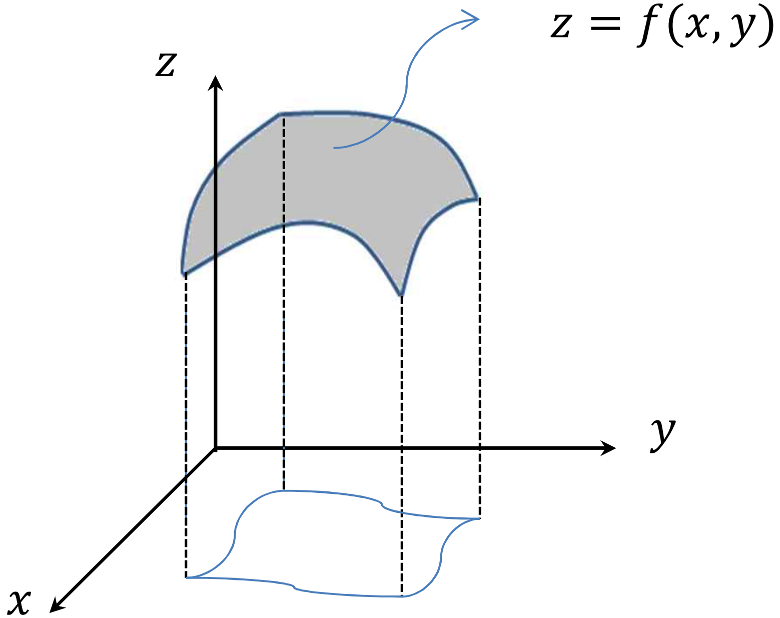
$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx = 2\int_0^\infty e^{-t^2}dt = \sqrt{\pi}$$

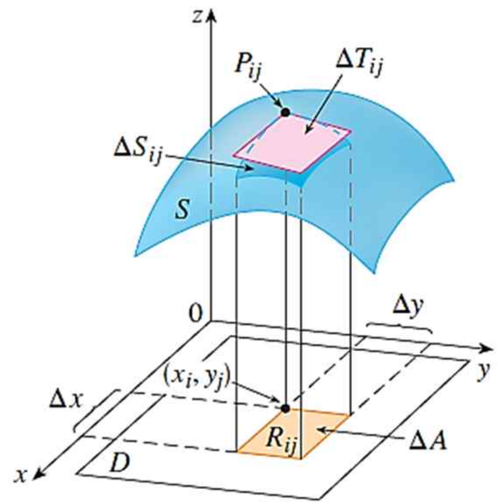
(4) $\sqrt{x}=t$로 치환하면 $\frac{1}{\sqrt{x}}dx = 2dt$이므로,

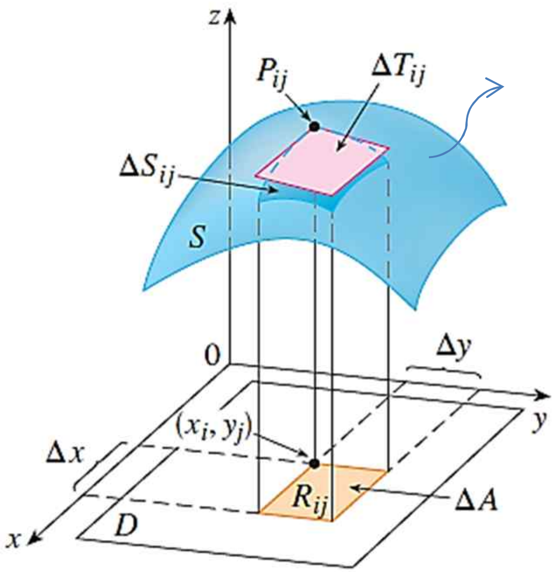
$$dx = 2\sqrt{x}dt = 2tdt$$

$$\begin{aligned}\int_0^\infty \sqrt{x}e^{-x}dx &= 2\int_0^\infty te^{-t^2}tdt\\ &= 2\int_0^\infty t^2e^{-t^2}dt\\ &= 2\frac{\sqrt{\pi}}{4} = \frac{\sqrt{\pi}}{2}\end{aligned}$$

15.4 곡면적


$z = f(x, y)$
z
y
x


z
P_{ij}
ΔT_{ij}
ΔS_{ij}
S
0
Δy
(x_i, y_j)
y
Δx
R_{ij}
ΔA
D
x

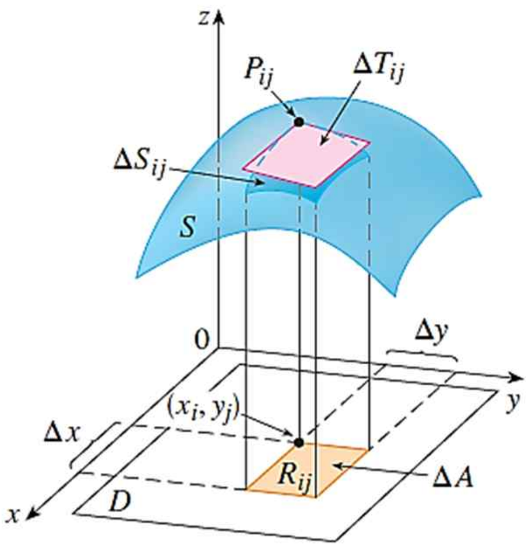


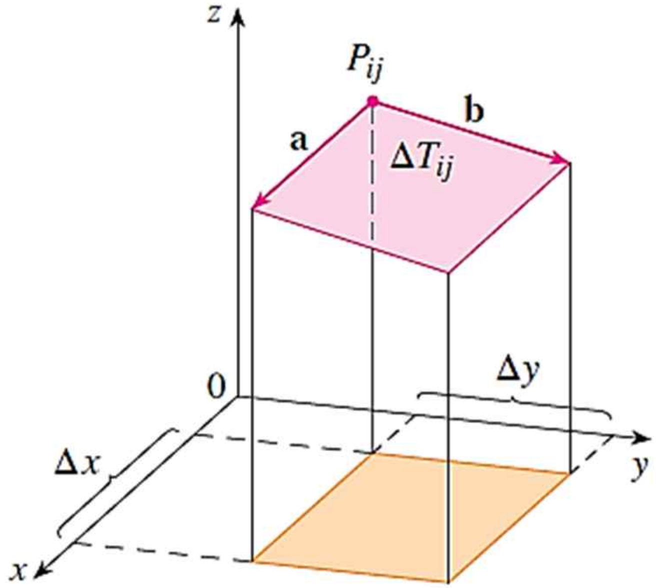
$z = f(x, y)$

곡면적을 S 라하면

$$S = \lim_{n,m\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \Delta T_{ij}$$

ΔT_{ij} 영역 R_{ij} 에서의 접평면의 넓이

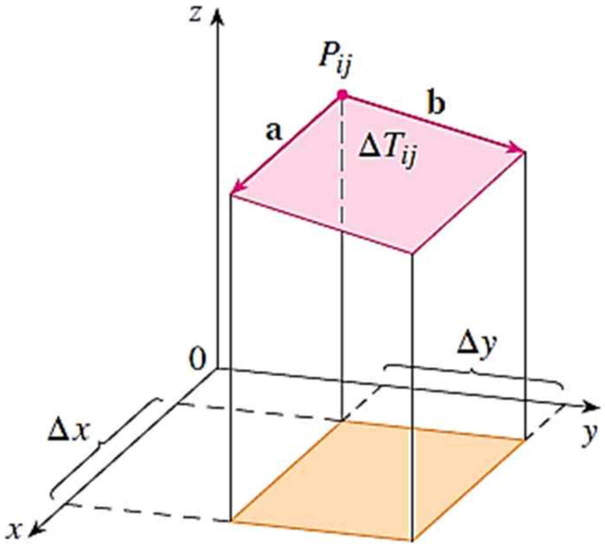


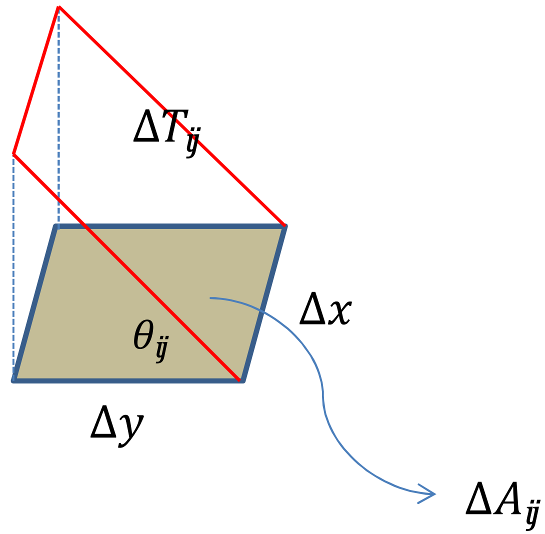


ΔT_{ij} 영역 R_{ij} 에서의 접평면의 넓이

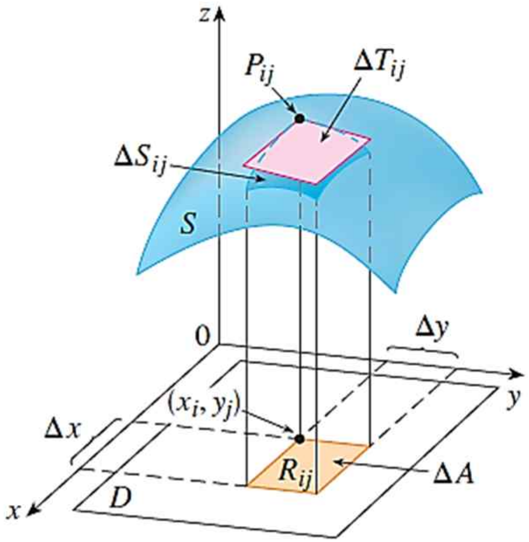
$$\Delta S_{ij} \approx \Delta T_{ij}$$

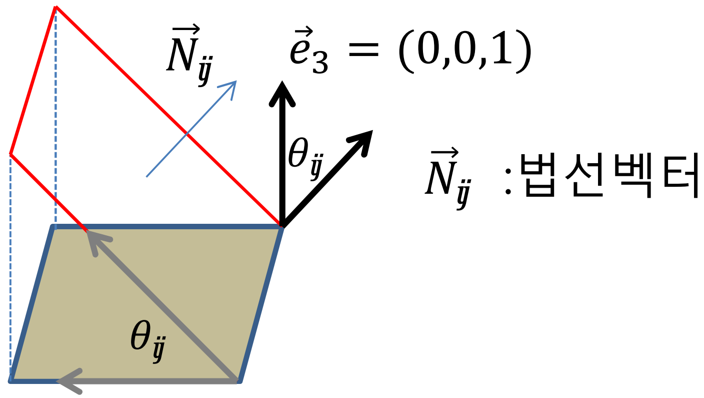
ΔS_{ij} 영역 R_{ij} 에서의 곡면의 넓이





$$\Delta T_{ij} = \frac{1}{\cos\theta_{ij}}\Delta A_{ij} = \frac{1}{\cos\theta_{ij}}\Delta x\Delta y$$

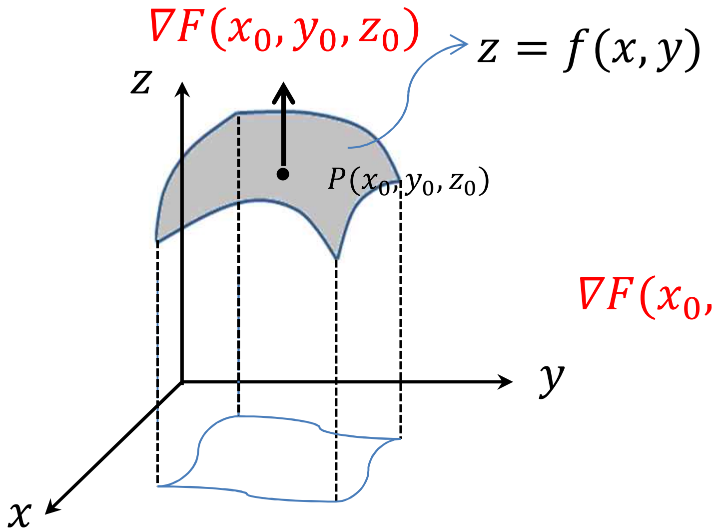




법선벡터는 접평면에 수직인 벡터

$$\cos\theta_{ij} = \frac{\vec{N}_{ij} \cdot \vec{e}_3}{|\vec{N}_{ij}||\vec{e}_3|}$$

$$\Delta T_{ij} = \frac{1}{\cos\theta_{ij}}\Delta A_{ij} = \frac{1}{\cos\theta_{ij}}\Delta x \Delta y$$



$$F(x, y, z) = f(x, y) - z = 0$$

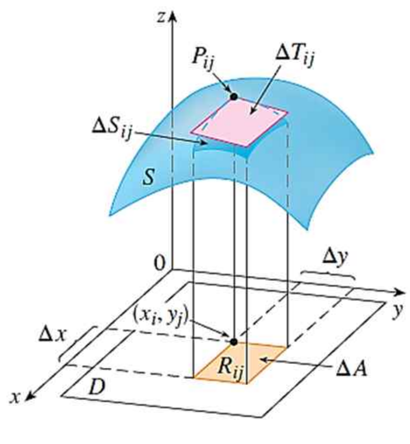
$\nabla F(x_0, y_0, z_0)$ 는 점 P 에서 곡면 의 법선벡터

$$\nabla F = (F_x, F_y, F_z) = (f_x, f_y, -1) = \vec{N}_{ij}$$

$$\cos\theta_{ij} = \frac{|\vec{N}_{ij} \cdot \vec{e}_3|}{|\vec{N}_{ij}||\vec{e}_3|} = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}$$

$$\Delta T_{ij} = \frac{1}{\cos\theta_{ij}} \Delta A_{ij} = \sqrt{f_x^2 + f_y^2 + 1}\, \Delta A_{ij}$$

$$\Delta T_{ij} = \frac{1}{\cos\theta_{ij}} \Delta A_{ij} = \sqrt{f_x^2 + f_y^2 + 1}\, \Delta A_{ij}$$



$$S = \lim_{n,m\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \Delta T_{ij}$$

$$= \lim_{n,m\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \sqrt{f_x^2 + f_y^2 + 1}\, \Delta A_{ij}$$

$$= \iint_{R_{xy}} \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$$z = f(x, y)$$

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

곡면적

적분영역이 xy-평면

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$z = f(x, y)$

적분영역이 yz-평면

$$S = \iint_{R_{yz}} \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1}\, dA$$

$x = f(y, z)$

적분영역이 xz-평면

$$S = \iint_{R_{xz}} \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1}\, dA$$

$y = f(x, z)$

적분영역이 xy-평면

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$z = f(x, y)$

적분영역이 yz-평면

$$S = \iint_{R_{yz}} \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1}\, dA$$

$x = f(y, z)$

적분영역이 xz-평면

$$S = \iint_{R_{xz}} \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1}\, dA$$

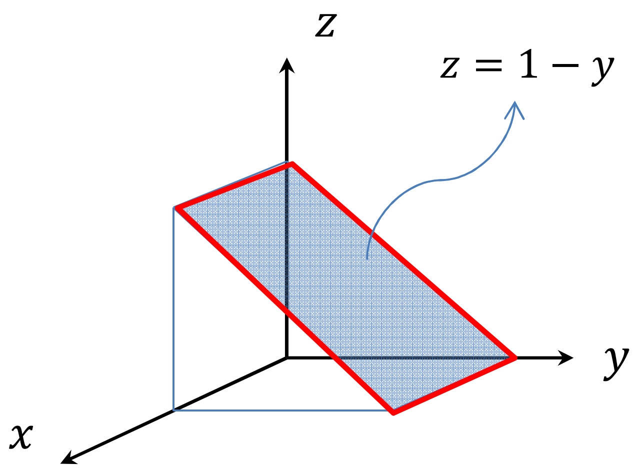
$y = f(x, z)$

Point: 적분영역을 결정하고 함수를 결정-부피 구할 때와 같이

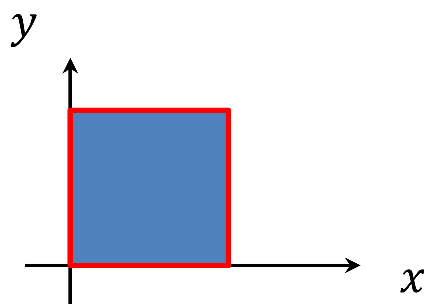
Example

제1팔분공간에서 평면 $z = 1 - y$ 를 두 평면 $x = 0, x = 1$ 에 의하여 잘린 부분의 곡면적을 구하여라

적분영역?

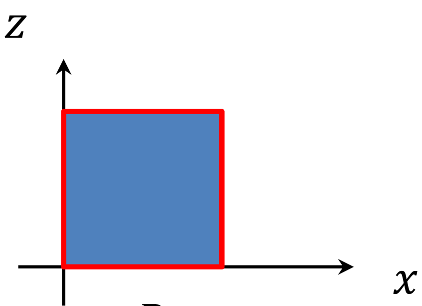


R_{yz}



R_{xy}

$z = 1 - y$

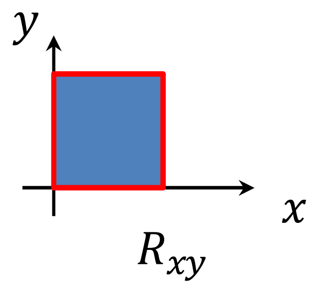


R_{xz}

$y = 1 - z$

$$S = \int_0^1 \int_0^1 \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dxdy$$

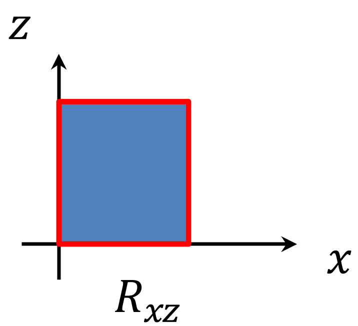
$$= \int_0^1 \int_0^1 \sqrt{2}\, dxdy \ = \sqrt{2}$$



$z = 1 - y$

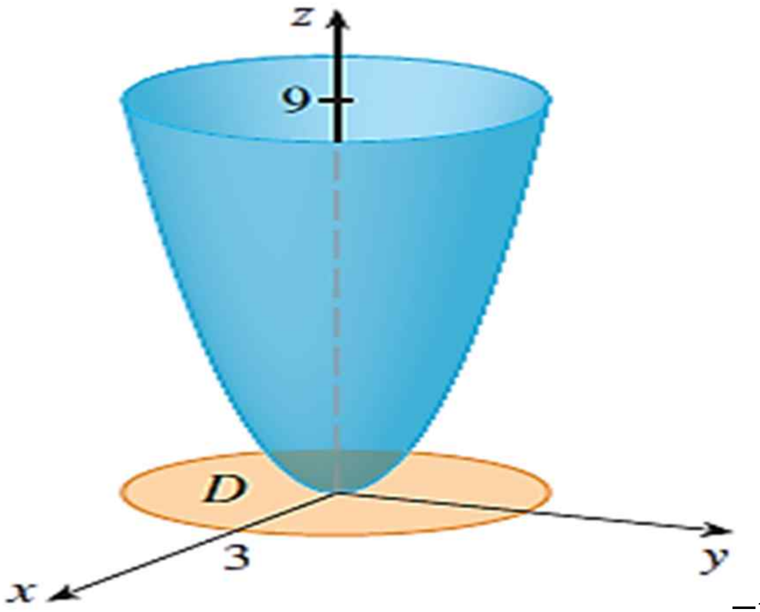
$$S = \int_0^1 \int_0^1 \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1}\, dydz$$

$$= \int_0^1 \int_0^1 \sqrt{2}\, dydz \ = \sqrt{2}$$



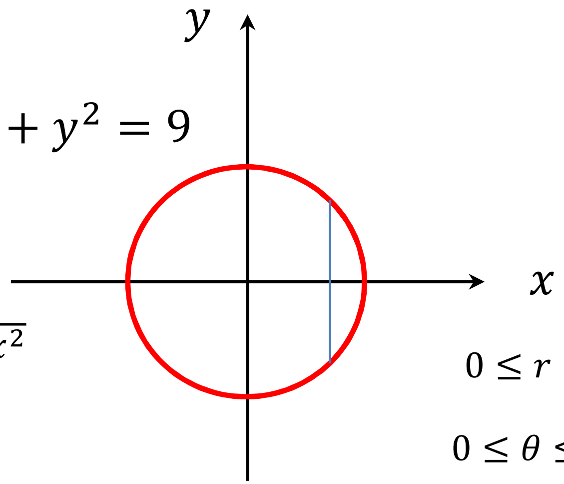
$y = 1 - z$

Example 평면 $z = 9$ 아래에 있는 원포물면 $z = x^2 + y^2$ 의 곡면적



적분영역

$x^2 + y^2 = 9$



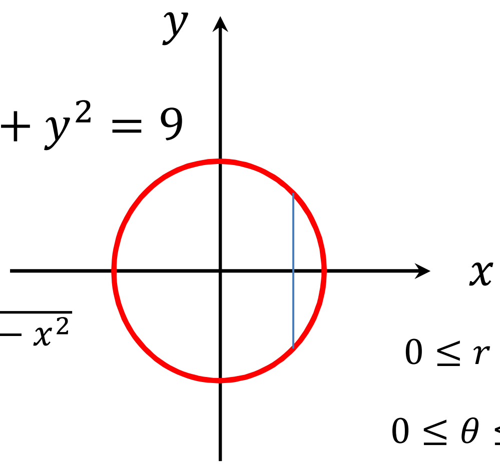
$-\sqrt{9 - x^2} \leq y \leq \sqrt{9 - x^2}$

$-3 \leq x \leq 3$

$0 \leq r \leq 3$

$0 \leq \theta \leq 2\pi$

$$x^2 + y^2 = 9$$



$$-\sqrt{9 - x^2} \le y \le \sqrt{9 - x^2}$$

$$-3 \le x \le 3$$

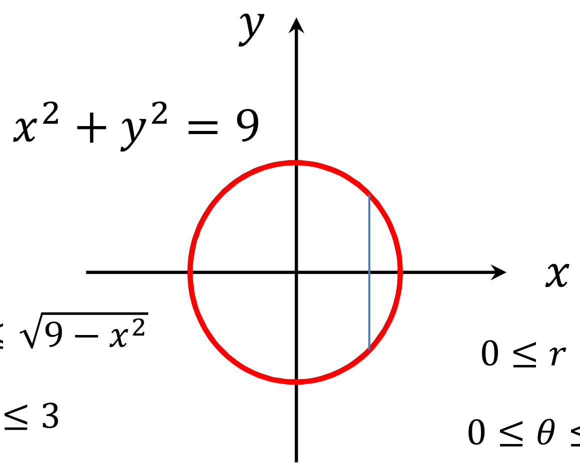
$$0 \le r \le 3$$

$$0 \le \theta \le 2\pi$$

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$$z = x^2 + y^2$$

$$\frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial y} = 2y$$



$$-\sqrt{9-x^2} \le y \le \sqrt{9-x^2}$$

$$-3 \le x \le 3$$

$$0 \le r \le 3$$

$$0 \le \theta \le 2\pi$$

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{4x^2 + 4y^2 + 1}\, dy\, dx$$

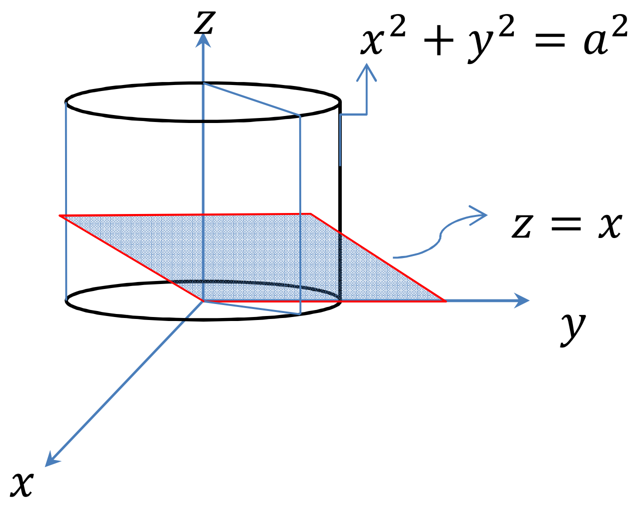
$$= \int_{0}^{2\pi} \int_{0}^{3} \sqrt{4r^2 + 1}\, r\, dr\, d\theta$$

$$= \int_{0}^{2\pi} \left[\frac{1}{8}\frac{2}{3}(4r^2+1)^{\frac{3}{2}}\right]_0^3 d\theta$$

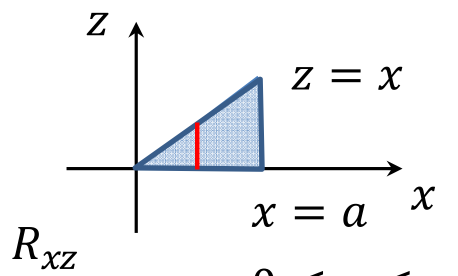
$$= \frac{\pi}{6}(37\sqrt{37} - 1)$$

Example

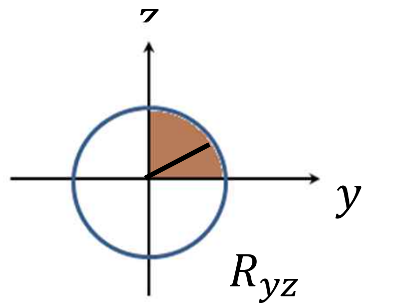
원기둥 $x^2 + y^2 = a^2$ 의 평면 $z = x$ 에 의하여 잘린 제1팔분공간에 있는 부분의 곡면적



적분영역



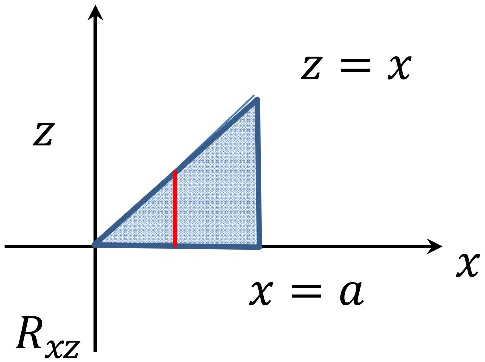
$0 \le z \le x$
$0 \le x \le a$



$0 \le y \le a$

$0 \le z \le \sqrt{a^2 - y^2}$

적분영역



$$0 \le z \le x$$

$$0 \le x \le a$$

$$y = \sqrt{a^2 - x^2}$$

$$\frac{\partial y}{\partial x} = \frac{-x}{\sqrt{a^2 - x^2}} \qquad \frac{\partial y}{\partial z} = 0$$

$$S = \int_0^a \int_0^x \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1 dzdx}$$

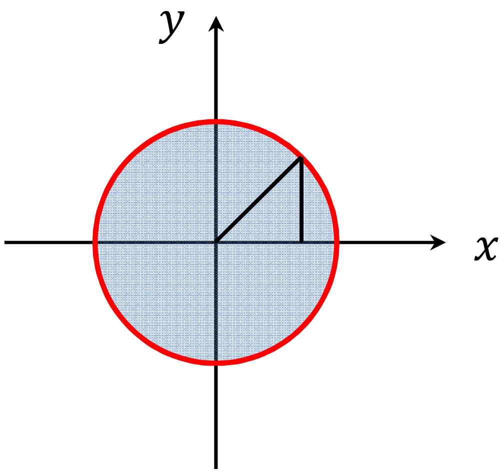
$$= \int_0^a \int_0^x \sqrt{\frac{x^2}{a^2 - x^2} + 1 dzdx}$$

$$= \int_0^a \int_0^x \frac{a}{\sqrt{a^2 - x^2}} dzdx$$

$$= a \int_0^a \frac{x}{\sqrt{a^2 - x^2}} dx \qquad = a\left[-\sqrt{a^2 - x^2}\right]_0^a = a^2$$

Example 구면 $x^2 + y^2 + z^2 = a^2$ 의 곡면적

적분영역



$$0 \le y \le \sqrt{a^2 - x^2}$$
$$0 \le x \le a$$

$$z = \sqrt{a^2 - x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{a^2 - x^2 - y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{-y}{\sqrt{a^2 - x^2 - y^2}}$$

$$0 \le r \le a$$
$$0 \le \theta \le \frac{\pi}{2}$$

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1dA}$$

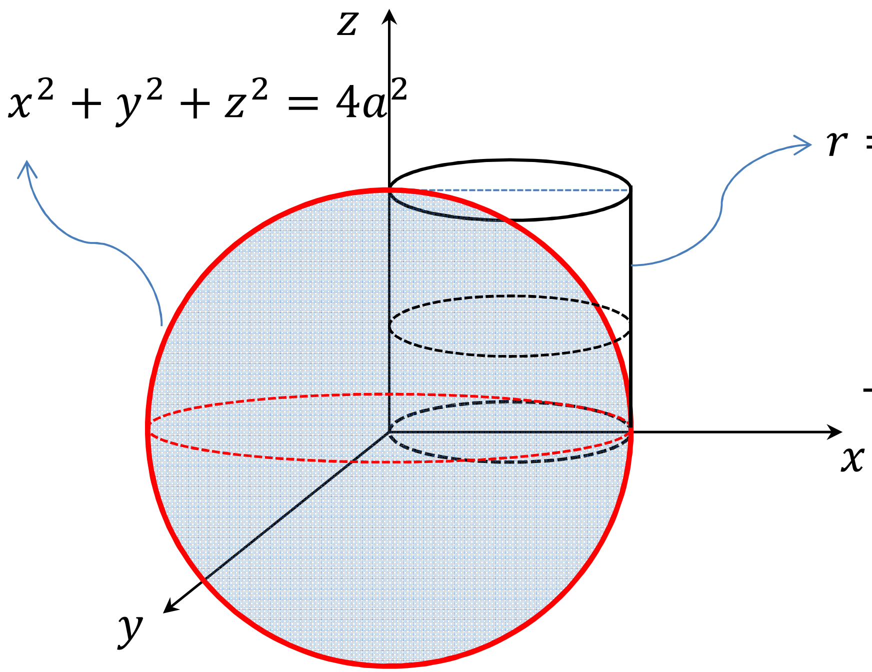
$$= 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{\frac{a^2}{a^2 - x^2 - y^2}}\,dydx$$

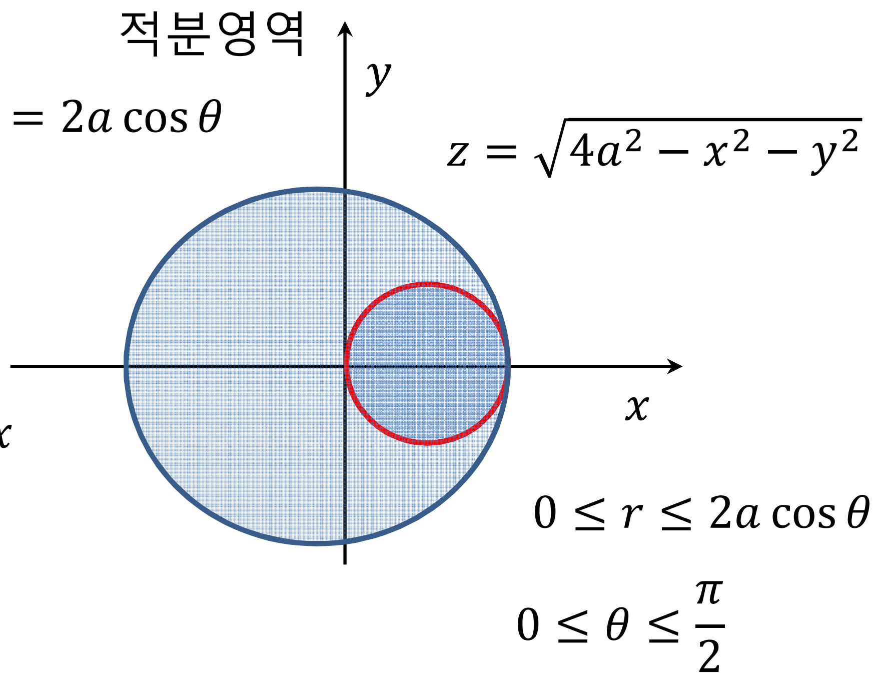
$$= 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{\frac{a^2}{a^2 - x^2 - y^2}}\,dydx$$

$$= 8a\int_0^{\frac{\pi}{2}} \int_0^a \sqrt{\frac{1}{a^2 - r^2}}\,rdrd\ \theta$$

$$= 8a\int_0^{\frac{\pi}{2}} \left[-\sqrt{a^2 - r^2}\right]_0^a d\theta \ = 4\pi a^2$$

Example 구면 $x^2+y^2+z^2=4a^2$ 에서 원기둥 $r=2a\cos\theta$ 의 내부에 있는 부분의 곡면적

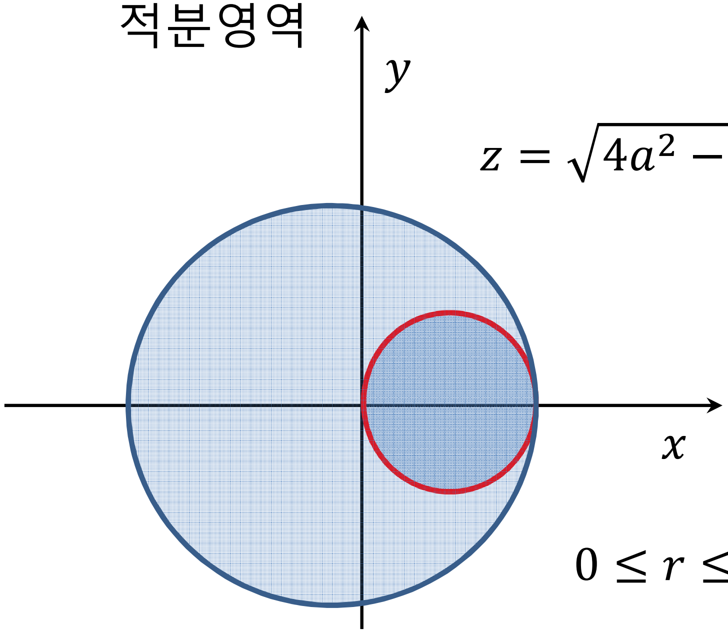




Example 구면 $x^2 + y^2 + z^2 = 4a^2$ 에서 원기둥 $r = 2a\cos\theta$ 의 내부에 있는 부분의 곡면적

$$S = \iint_{R_{xy}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

적분영역



$$z = \sqrt{4a^2 - x^2 - y^2}$$

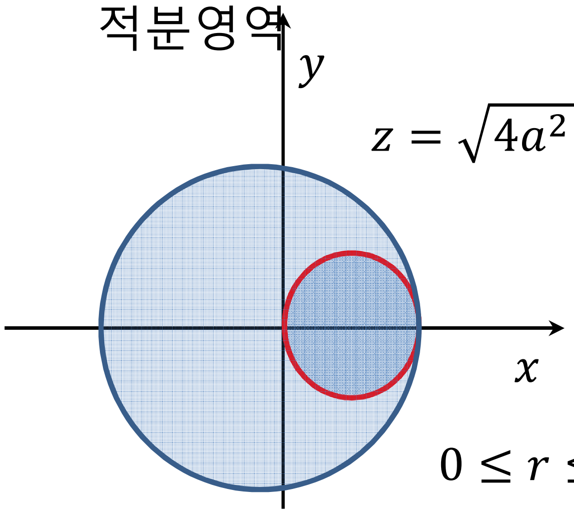
$$= 4\iint_R \frac{2a}{\sqrt{4a^2 - x^2 - y^2}}\, dA$$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{4a^2 - x^2 - y^2}}$$

$$0 \le r \le 2a\cos\theta$$

$$0 \le \theta \le \frac{\pi}{2}$$

$$\frac{\partial z}{\partial y} = \frac{-y}{\sqrt{4a^2 - x^2 - y^2}}$$



$$z = \sqrt{4a^2 - x^2 - y^2}$$

$$0 \le r \le 2a\cos\theta$$

$$0 \le \theta \le \frac{\pi}{2}$$

$$S = 4\iint_R \frac{2a}{\sqrt{4a^2 - x^2 - y^2}}\,dA$$

$$= 8a\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} \frac{r\,dr\,d\theta}{\sqrt{4a^2 - r^2}}$$

$$= -8a\int_0^{\frac{\pi}{2}}\left[\sqrt{4a^2 - r^2}\right]_0^{2a\cos\theta} d\theta$$

$$= -8a\int_0^{\frac{\pi}{2}}(a\sin\theta - 2a)\,d\theta$$

$$= 8a^2(\pi - 2)$$