CHAPTER 4

Applications of Boolean Algebra/ Minterm and Maxterm Expansions

This chapter in the book includes:

Objective

- Conversion of English Sentences to Boolean Equations
- Combinational Logic Design Using a Truth Table
- Minterm and Maxterm Expansions
- General Minterm and Maxterm Expansions
- Incompletely Specified Functions (Don’t care term)
- Examples of Truth Table Construction
- Design of Binary Adders(Full adder) and Subtracters

4.1 Conversion of English Sentences to Boolean Equations

- Steps in designing a single-output combinational switching circuit

1. Find switching function which specifies the desired behavior of the circuit
2. Find a simplified algebraic expression for the function
3. Realize the simplified function using available logic elements

1. F is 'true' if A and B are both 'true' → F=AB

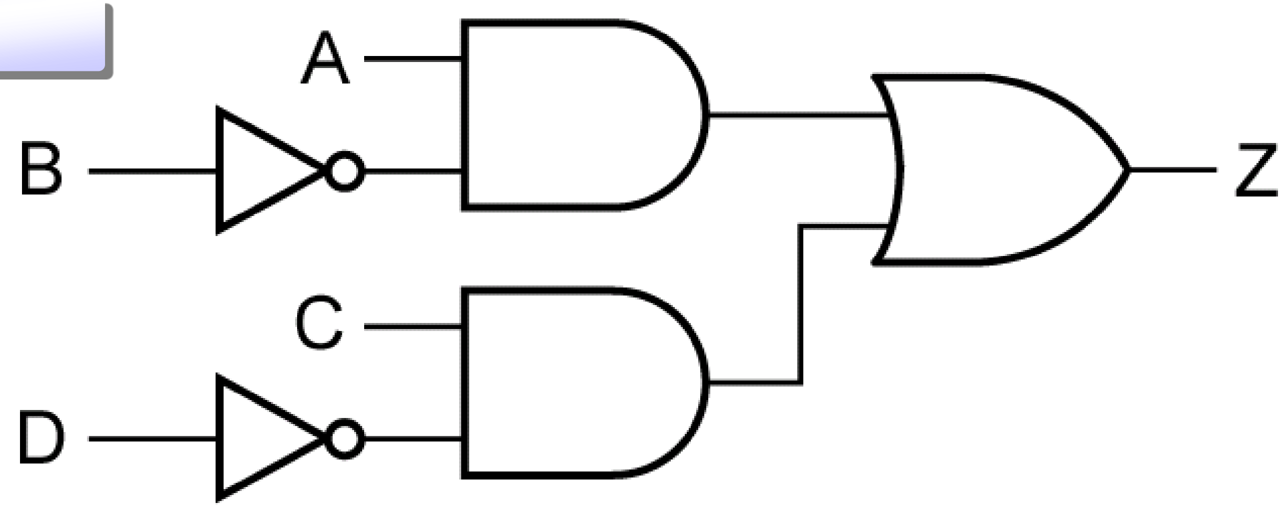
4.1 Conversion of English Sentences to Boolean Equations

1. The alarm will ring(Z) iff the alarm switch is turned on(A) ***and*** the door is not closed(B'), ***or*** it is after 6PM(C) and window is not closed(D')

2. Boolean Equation

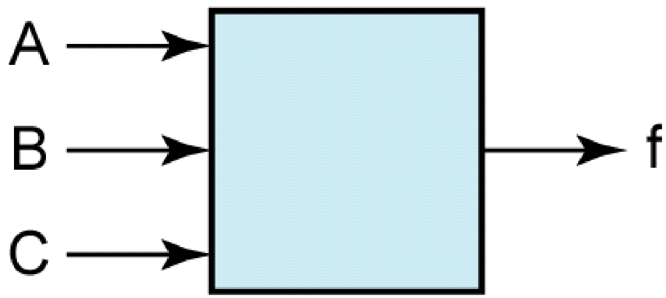
$$Z = AB' + CD'$$

3. Circuit realization



4.2 Combinational Logic Design Using a Truth Table

- Combinational Circuit with Truth Table



(a)

A	B	C	f	f'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

(b)

When expression for *f=1* →

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

4.2 Combinational Logic Design Using a Truth Table

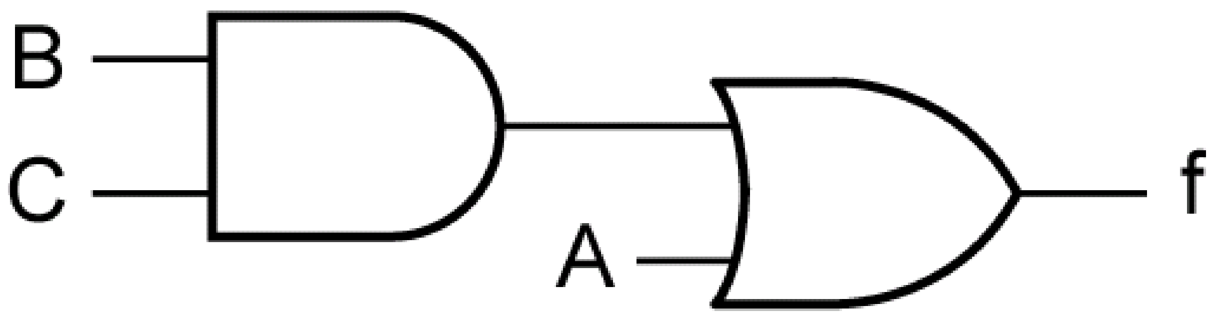
Original equation →

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

Simplified equation →

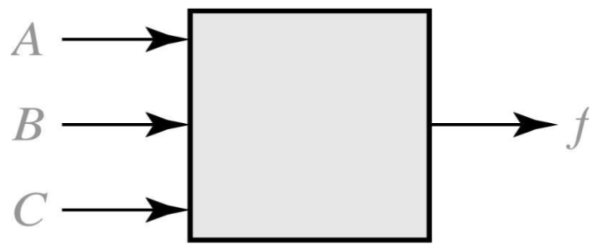
$$f = A'BC + AB' + AB = A'BC + A = A + BC$$

Circuit realization →



4.2 Combinational Logic Design Using a Truth Table

- Combinational Circuit with Truth Table



(a)

A	B	C	f	f'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

(b)

When expression for *f=0* →

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

$$f = (A+B)(A+B'+C) = A + B(B'+C) = A + BC$$

When expression for *f '=1* → and take the complement of *f '* →

$$f' = A'B'C' + A'B'C + A'BC'$$

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

4.3 Minterm and Maxterm Expansions

- *literal* is a variable or its complement (e.g. *A, A'*)

- *Minterm, Maxterm* for three variables

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

4.3 Minterm and Maxterm Expansions

- *Minterm* of *n* variables is a product of *n* literals in which each variable appears exactly once in either true (*A*) or complemented form(*A'*), but not both. (→ m_0)

-Minterm expansion,
-Standard Sum of Product →

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$$

$$f(A,B,C) = \sum m(3,4,5,6,7)$$

4.3 Minterm and Maxterm Expansions

- *Maxterm* of *n* variables is a sum of *n* literals in which each variable appears exactly once in either true (*A*) or complemented form(*A'*) , but not both.(→ M_0)

-Maxterm expansion,
-Standard Product of Sum→

$$f = (A+B+C)(A+B+C')(A+B'+C)$$

$$f(A,B,C) = M_0 M_1 M_2$$

$$f(A,B,C) = \prod M(0,1,2)$$

4.3 Minterm and Maxterm Expansions

$$f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$$

$$f' = m_0 + m_1 + m_2 = \sum m(0,1,2)$$

$$f(A,B,C) = M_0 M_1 M_2 \longrightarrow f' = \prod M(3,4,5,6,7) = M_3 M_4 M_5 M_6 M_7$$

- *Minterm* and *Maxterm* expansions are complement each other

$$f' = (m_3 + m_4 + m_5 + m_6 + m_7)' = m'_3\, m'_4\, m'_5\, m'_6\, m'_7 = M_3 M_4 M_5 M_6 M_7$$

$$f' = (M_0 M_1 M_2)' = M'_0 + M'_1 + M'_2 = m_0 + m_1 + m_2$$

4.3 Minterm and Maxterm Expansions

-Example: *Minterm* expansion

$$
\begin{aligned}
f &= a'b' + a'd + acd' \\
&= a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b') \\
&= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + \cancel{a'b'c'd} + \cancel{a'b'cd} \\
&\qquad + a'bc'd + a'bcd + abcd' + ab'cd' \qquad (4\text{-}9)
\end{aligned}
$$

$$
\begin{array}{l}
f = a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bc'd + a'bcd + abcd' + ab'cd' \\
\quad 0\,0\,0\,0 \qquad 0\,0\,0\,1 \qquad 0\,0\,1\,0 \qquad 0\,0\,1\,1 \qquad 0\,1\,0\,1 \qquad 0\,1\,1\,1 \qquad 1\,1\,1\,0 \qquad 1\,0\,1\,0
\end{array}
$$

$$f = \Sigma\, m(0, 1, 2, 3, 5, 7, 10, 14) \qquad (4\text{-}10)$$

4.3 Minterm and Maxterm Expansions

-Example: *Maxterm* expansion

$$
\begin{aligned}
f &= a'(b' + d) + acd' \\
&= (a' + cd')(a + b' + d) = (a' + c)(a' + d')(a + b' + d) \\
&= (a' + bb' + c + dd')(a' + bb' + cc' + d')(a + b' + cc' + d) \\
&= (a' + bb' + c + d)(a' + bb' + c + d')\cancel{(a' + bb' + c + d')} \\
&\qquad (a' + bb' + c' + d')(a + b' + cc' + d) \\
&= \underset{1000}{(a' + b + c + d)}\underset{1100}{(a' + b' + c + d)}\underset{1001}{(a' + b + c + d')}\underset{1101}{(a' + b' + c + d')} \\
&\qquad \underset{1011}{(a' + b + c' + d')}\underset{1111}{(a' + b' + c' + d')}\underset{0100}{(a + b' + c + d)}\underset{0110}{(a + b' + c' + d)} \\
&= \Pi\, M(4, 6, 8, 9, 11, 12, 13, 15) \qquad (4\text{-}11)
\end{aligned}
$$

4.4 General Minterm and Maxterm Expansions

A B C	F
0 0 0	a_0
0 0 1	a_1
0 1 0	a_2
0 1 1	a_3
1 0 0	a_4
1 0 1	a_5
1 1 0	a_6
1 1 1	a_7

- General truth table for 3 variables
- a_i is either '0' or '1'

- Minterm expansion for general function

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \ldots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i$$

a_i =1, minterm m_i is present

a_i =0, minterm m_i is not present

- Maxterm expansion for general function

$$F = (a_0 + M_0)(a_1 + M_1)(a_2 + M_2)\ldots(a_7 + M_7) = \prod_{i=0}^{7} (a_i + M_i)$$

a_i =1, $a_i + M_i$ =1 , Maxterm M_i is not present

a_i =0, Maxterm is present

4.4 General Minterm and Maxterm Expansions

$$F' = [\prod_{i=0}^{7}(a_i + M_i)]' = \sum_{i=0}^{7} a'_i M'_i = \sum_{i=0}^{7} a'_i m_i$$

→ All minterm which are not present in *F* are present in *F* ‘

$$F' = [\sum_{i=0}^{7} a_i m_i]' = \prod_{i=0}^{7}(a'_i + m'_i) = \prod_{i=0}^{7}(a'_i + M_i)$$

→ All maxterm which are not present in *F* are present in *F* ‘

$$F = \sum_{i=0}^{2^n-1} a_i m_i = \prod_{i=0}^{2^n-1}(a_i + M_i)$$

$$F' = \sum_{i=0}^{2^n-1} a'_i m_i = \prod_{i=0}^{2^n-1}(a'_i + M_i)$$

4.4 General Minterm and Maxterm Expansions

If i and j are different, $m_i m_j = 0$

$$f_1 = \sum_{i=0}^{2^n-1} a_i m_i \qquad f_2 = \sum_{j=0}^{2^n-1} b_j m_j$$

$$f_1 f_2 = \left(\sum_{i=0}^{2^n-1} a_i m_i\right)\left(\sum_{j=0}^{2^n-1} b_j m_j\right) = \sum_{i=0}^{2^n-1}\sum_{j=0}^{2^n-1} a_i b_j m_i m_j = \sum_{i=0}^{2^n-1} a_i b_i m_i$$

Example:

$f_1 = \sum m(0,2,3,5,9,11)$ and $f_2 = m(0,3,9,11,13,14)$

$$f_1 f_2 = \sum m(0,3,9,11)$$

Conversion between minterm and maxterm expansions of *F* and *F'*

Conversion of forms

GIVEN FORM / DESIRED FORM

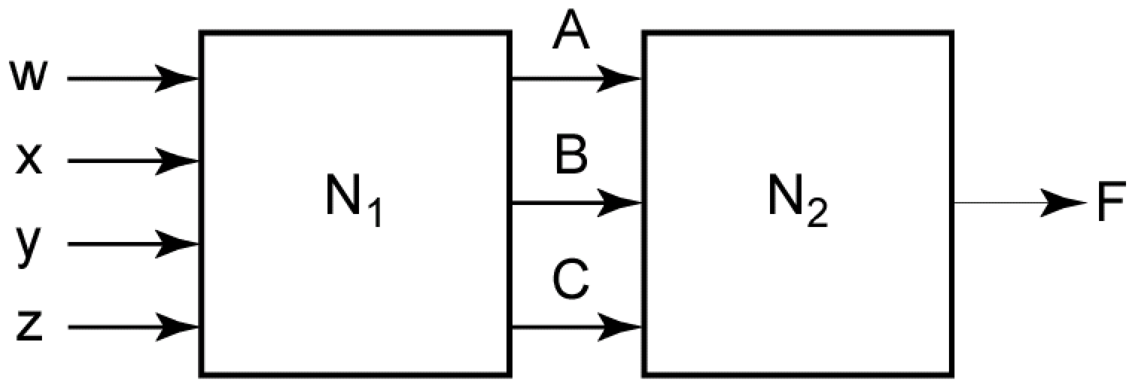
	Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F'	Maxterm Expansion of F'
Minterm Expansion of F	――――	maxterm nos. are those nos. not on the minterm list for F	list minterms not present in F	maxterm nos. are the same as minterm nos. of F
Maxterm Expansion of F	minterm nos. are those nos. not on the maxterm list for F	――――	minterm nos. are the same as maxterm nos. of F	list maxterms not present in F

Application of Table4.3

GIVEN FORM / DESIRED FORM

	Minterm Expansion of f	Maxterm Expansion of f	Minterm Expansion of f'	Maxterm Expansion of f'
$f = \Sigma m(3, 4, 5, 6, 7)$	――――	$\Pi M(0, 1, 2)$	$\Sigma m(0, 1, 2)$	$\Pi M(3, 4, 5, 6, 7)$
$f = \Pi M(0, 1, 2)$	$\Sigma m(3, 4, 5, 6, 7)$	――――	$\Sigma m(0, 1, 2)$	$\Pi M(3, 4, 5, 6, 7)$

4.5 Incompletely Specified Functions



If N_1 output does not generate all possible combination of A,B,C, the output of N_2(F) has 'don't care' values.

Truth Table with Don't Cares

A B C	F
0 0 0	1
0 0 1	X
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	X
1 1 1	1

4.5 Incompletely Specified Functions

Finding Function:

Case 1: assign ‘0’ on X’s

$$F = A'B'C' + A'BC + ABC = A'B'C' + BC$$

Case 2: assign ‘1’ to the first X and ‘0’ to the second ‘X’

$$F = A'B'C' + A'B'C + A'BC + ABC = A'B' + BC$$

Case 3: assign ‘1’ on X’s

$$F = A'B'C' + A'B'C + \underline{A'BC} + ABC' + \underline{ABC} = A'B' + \underline{BC} + AB$$

→ The case 2 leads to the simplest function

4.5 Incompletely Specified Functions

Minterm expansion for incompletely specified function

$$F = \sum m(0,3,7) + \sum d(1,6)$$

Don't Cares

Maxterm expansion for incompletely specified function

$$F = \prod M(2,4,5) \cdot \prod D(1,6)$$

4.6 Examples of Truth Table Construction

Example 1 : Binary Adder

a	b	Sum	
0	0	0 0	0+0=0
0	1	0 1	0+1=1
1	0	0 1	1+0=1
1	1	1 0	1+1=2

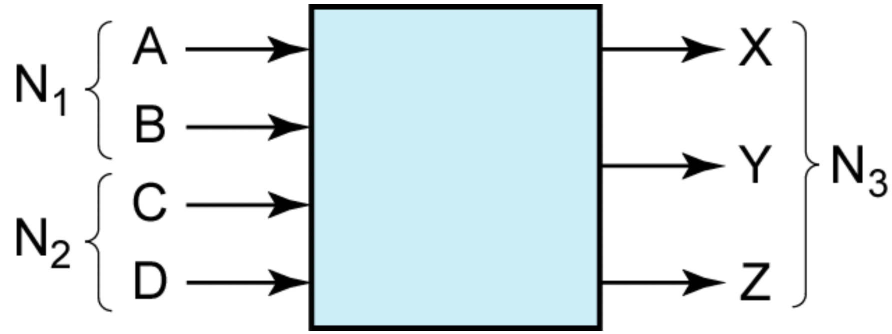
→

A	B	X	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$X = AB, Y = A'B + AB' = A \oplus B$$

4.6 Examples of Truth Table Construction

Example 2 : 2 bit binary Adder



TRUTH TABLE:

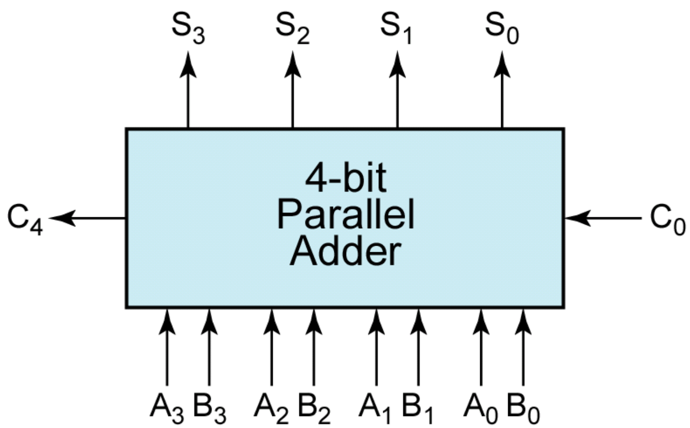
N_1		N_2		N_3		
A	B	C	D	X	Y	Z
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0

TRUTH TABLE:

N_1		N_2		N_3		
A	B	C	D	X	Y	Z
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

4.7 Design of Binary Adders and Subtracters

Parallel Adder for 4 bit Binary Numbers

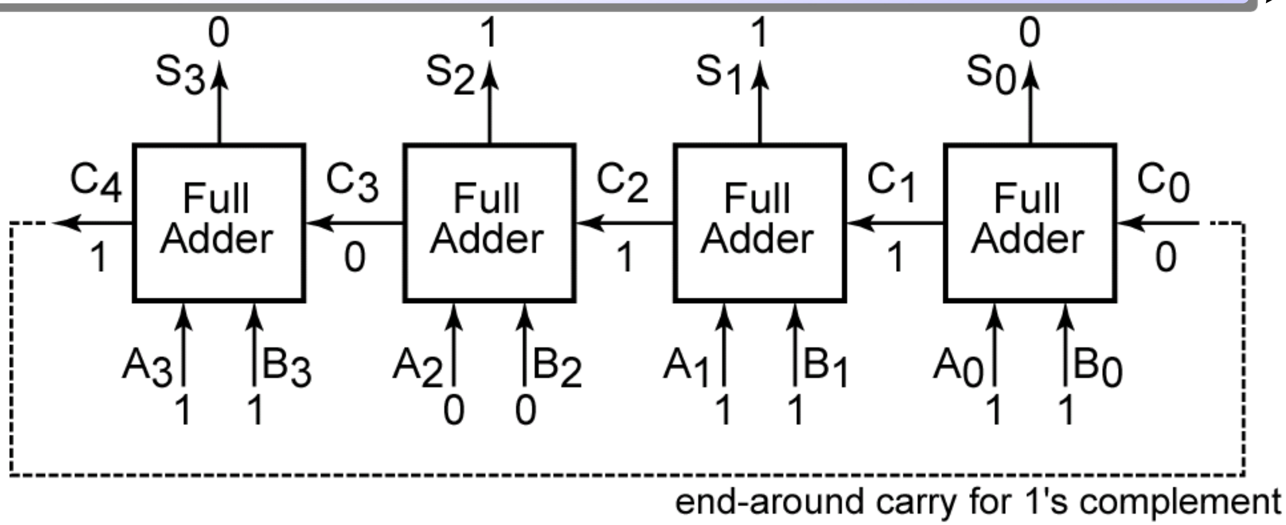


10110 (carries)

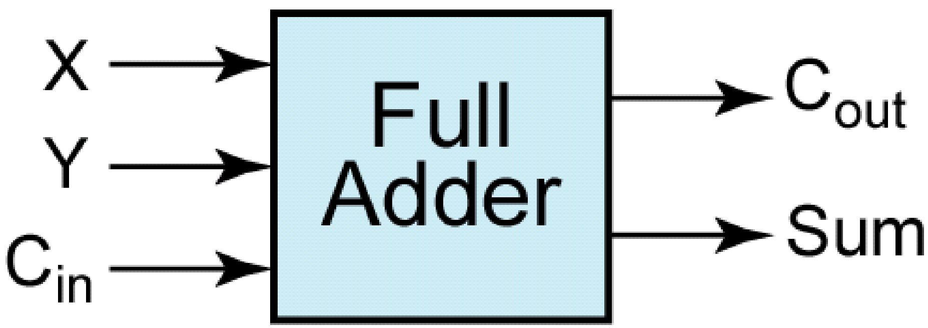
  1011
+1011

10110

Parallel adder composed of four full adders ← Carry Ripple Adder (slow!)



4.7 Design of Binary Adders and Subtracters



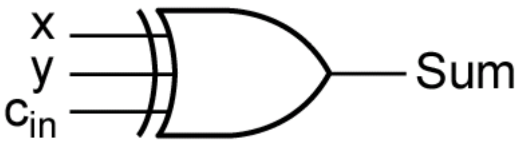
Truth Table for a Full Adder

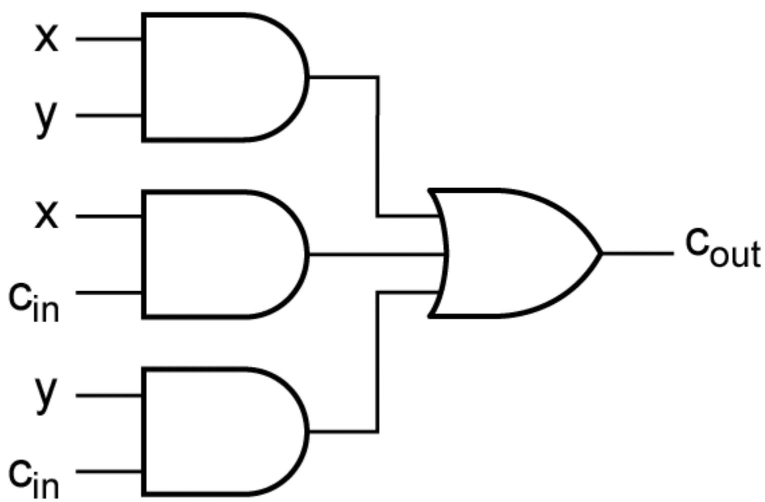
X	Y	C_{in}	C_{out}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

4.7 Design of Binary Adders and Subtracters

$$
\begin{aligned}
Sum &= X'Y'C_{in} + X'YC'_{in} + XY'C'_{in} + XYC_{in} \\
&= X'(Y'C_{in} + YC'_{in}) + X(Y'C'_{in} + YC_{in}) \\
&= X'(Y \oplus C_{in}) + X(Y \oplus C_{in})' = X \oplus Y \oplus C_{in}
\end{aligned}
$$

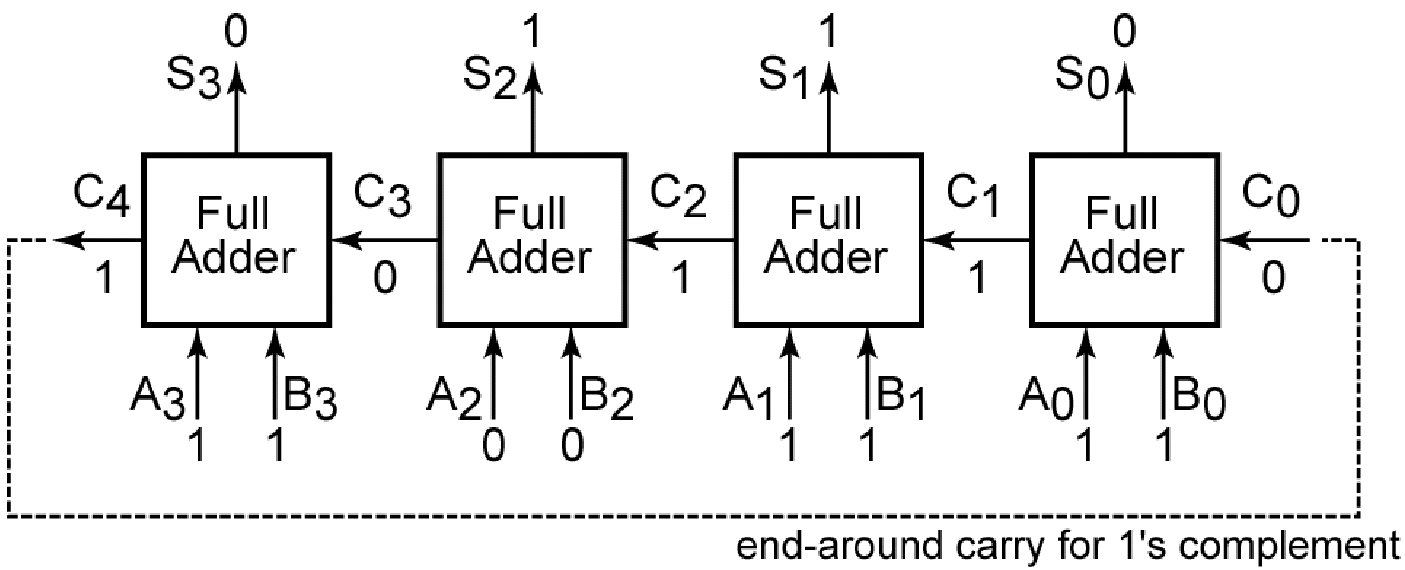
$$
\begin{aligned}
C_{out} &= X'YC_{in} + XY'C_{in} + XYC'_{in} + \underline{XYC_{in}} \\
&= (X'YC_{in} + \underline{XYC_{in}}) + (XY'C_{in} + \underline{XYC_{in}}) + (XYC'_{in} + \underline{XYC_{in}}) \\
&= YC_{in} + XC_{in} + XY
\end{aligned}
$$





4.7 Design of Binary Adders and Subtracters

When 1's complement is used, the end-around carry is accomplished by connecting C_4 to C_0 input.



Overflow(V) when adding two signed binary number

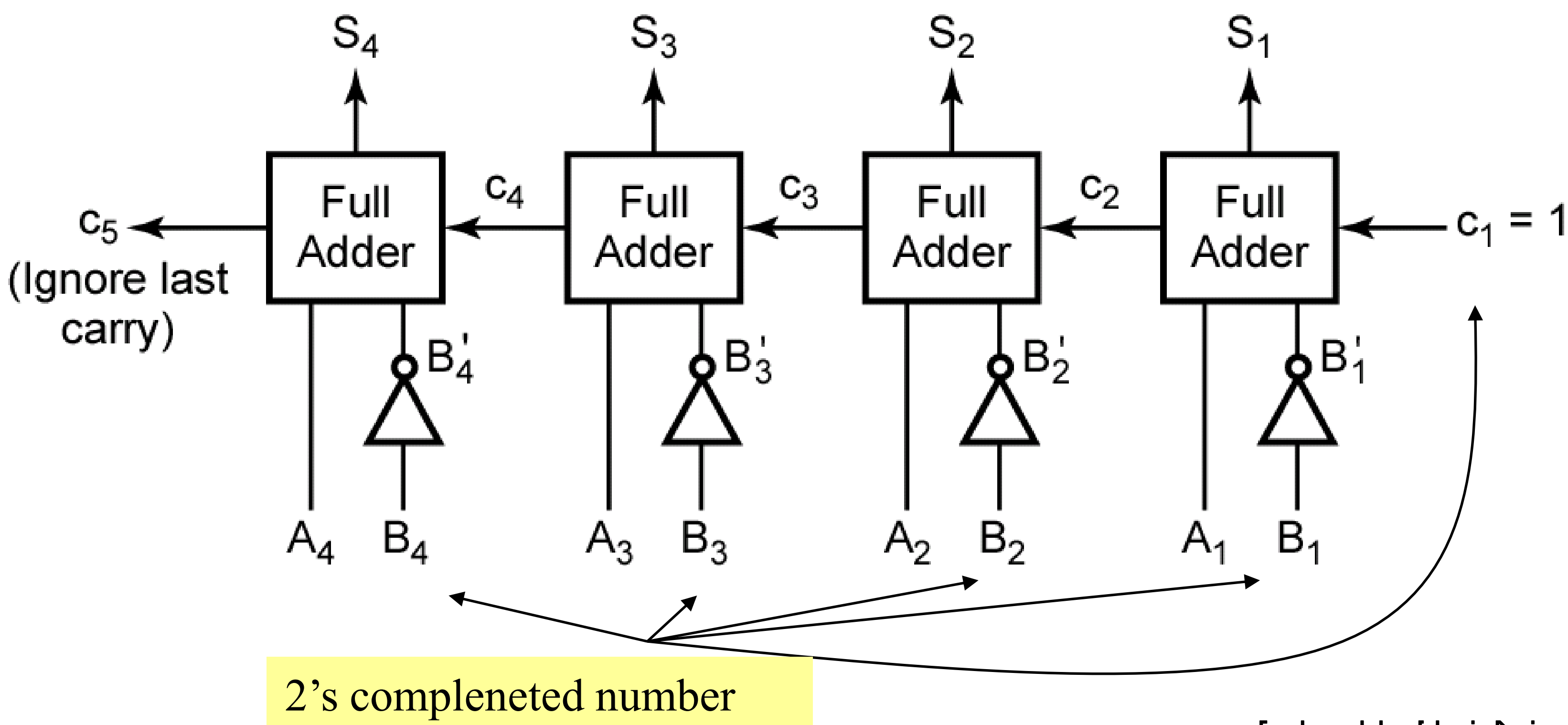
$$V = A'_3 B'_3 S_3 + A_3 B_3 S'_3$$

4.7 Design of Binary Adders and Subtracters

Subtracters

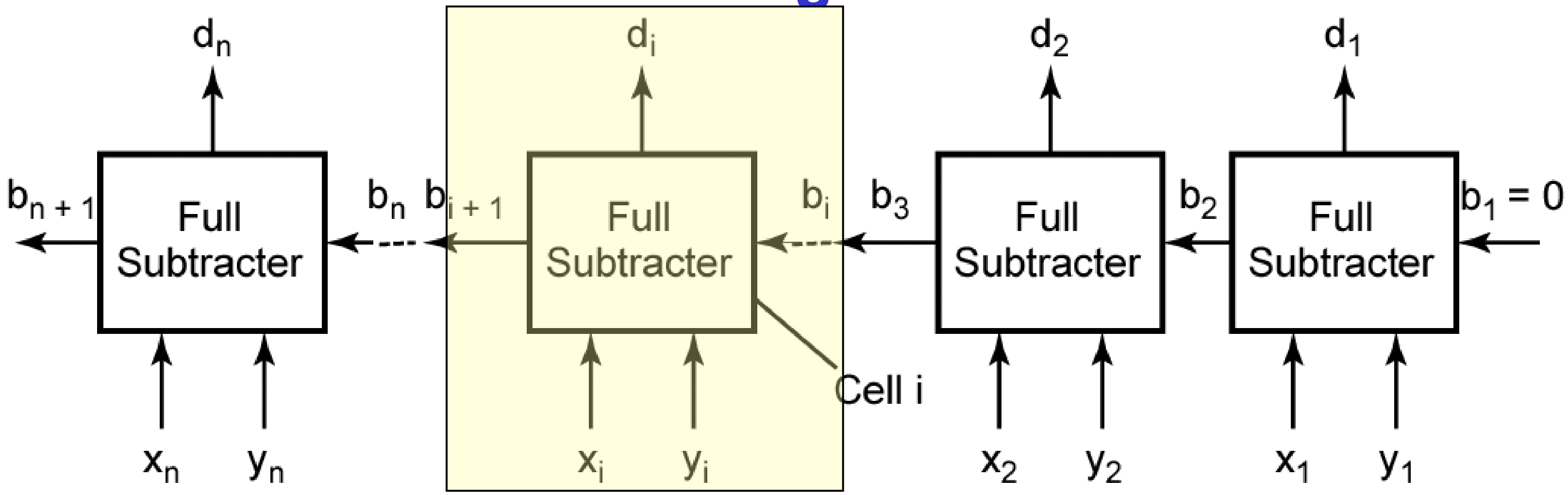
Binary Subtracter using full adder

- Subtraction is done by adding the 2's complemented number to be subtracted



4.7 Design of Binary Adders and Subtracters

Subtracters- using Full Subtracter



Truth Table for a Full Subtracter

x_i	y_i	b_i	b_{i+1}	d_i
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1