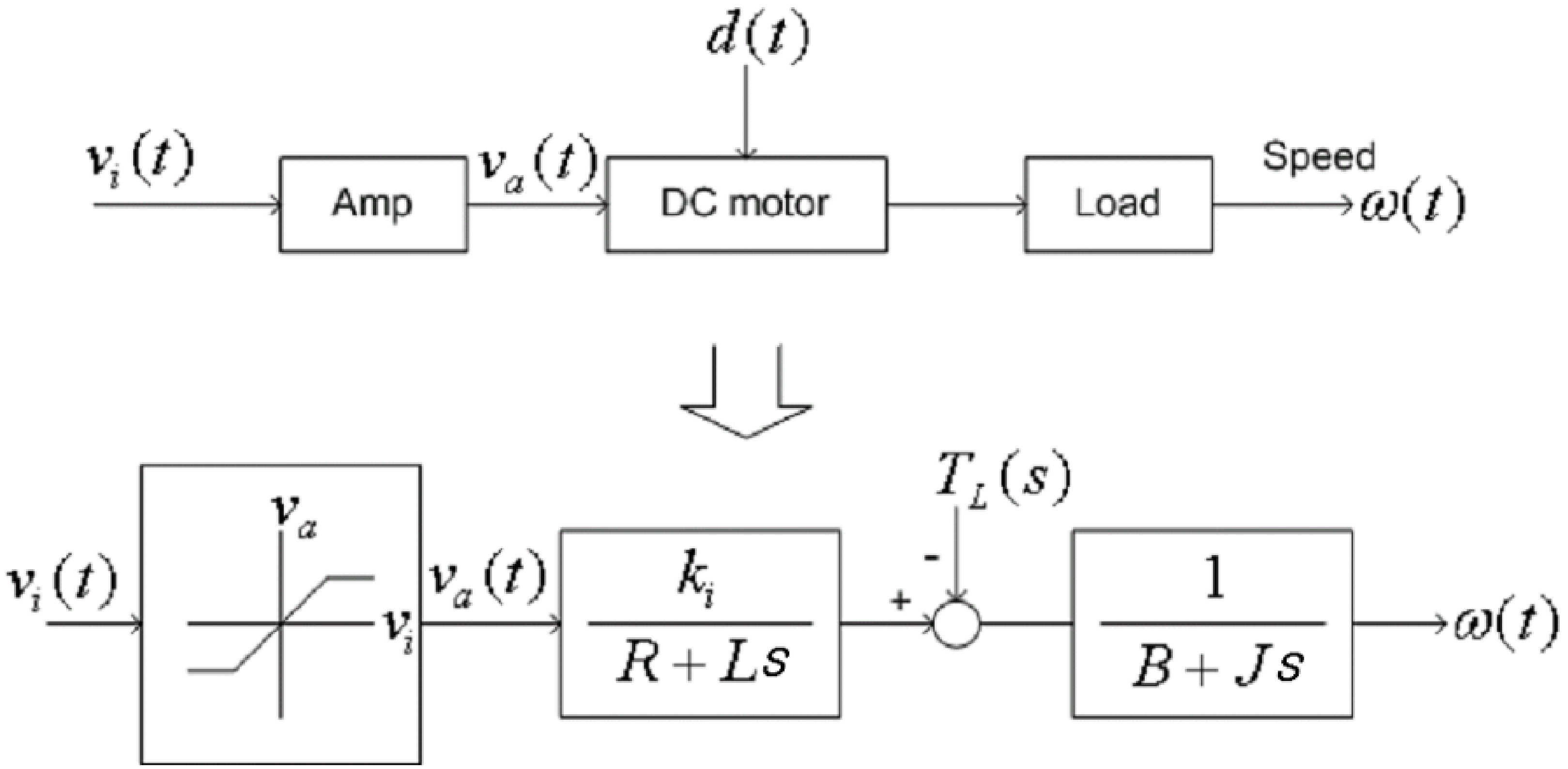
Automatic Control Systems
-Lecture Note 8-

Block Diagram and Signal-Flow Graph 2

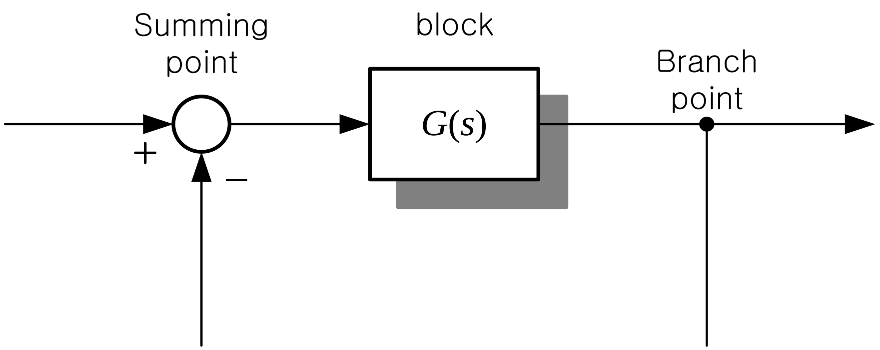
Block Diagram

[Example] DC motor control system



Block Diagram

- Components
  - **block** : represents input-output system of transfer function
  - **summing point** : signals are added. + for addition and – for subtraction are used
  - **branch point** : departure point of signals

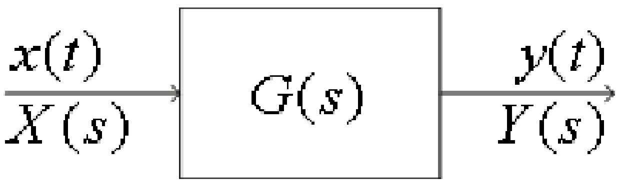


<Fig> Components of block diagram

Block Diagram

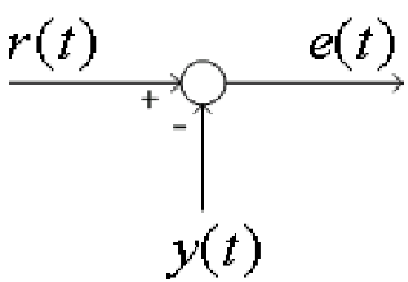
- Components of block diagram

1) System Block



$$Y(s) = G(s) \cdot X(s)$$

2) Adder(or Subtracter)

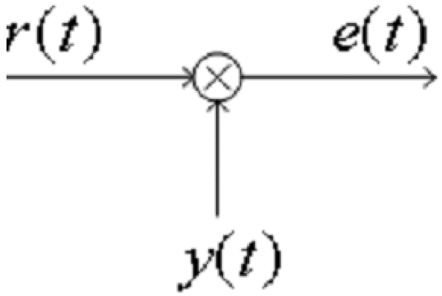


$$e(t) = r(t) - y(t)$$

$$(\ E(s) = R(s) - Y(s) \ )$$

Block Diagram

3) Multiplier



$$e(t) = r(t) \cdot y(t)$$

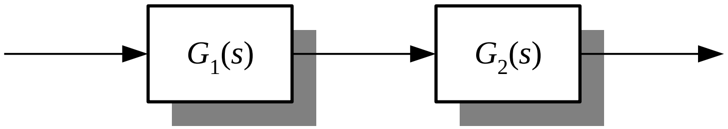
$$(\ E(s) \neq R(s) \cdot Y(s)\ )$$

Block Diagram

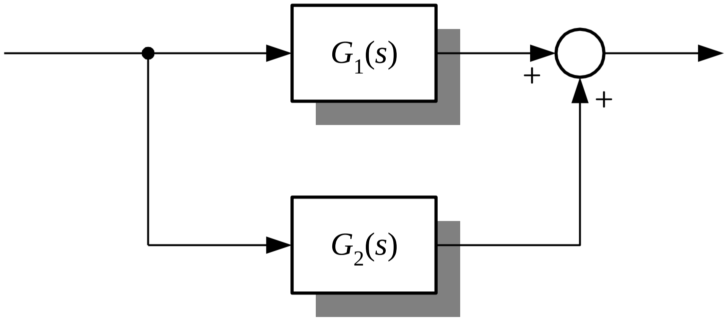
Basic connection

- serial : $G_s(s) = G_1(s)G_2(s)$
- parallel : $G_p(s) = G_1(s) + G_2(s)$
- feedback : $G_c(s) = \dfrac{G_1(s)}{1 + G_1(s)G_2(s)}$

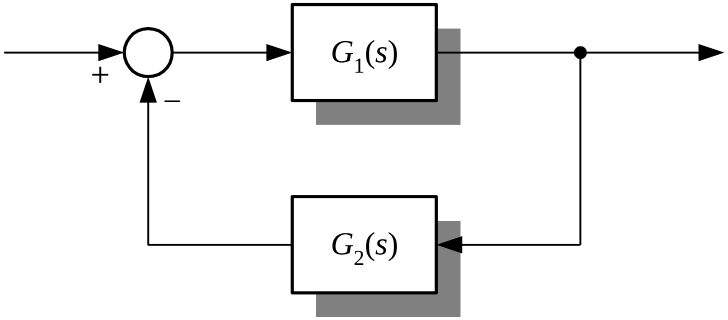
Block Diagram



(a) $G_s(s) = G_1(s)G_2(s)$



(b) $G_p(s) = G_1(s)+G_2(s)$

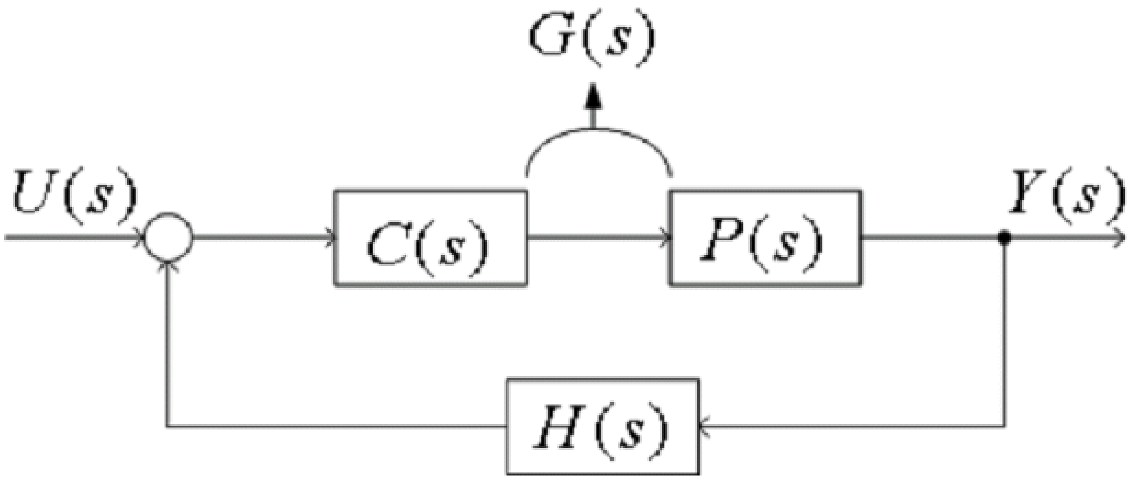


(c) $G_c(s) = \dfrac{G_1(s)}{1+G_1(s)G_2(s)}$

<Fig> Basic Connection

Block Diagram

[Example]



$$Y(s) = M(s)U(s)$$

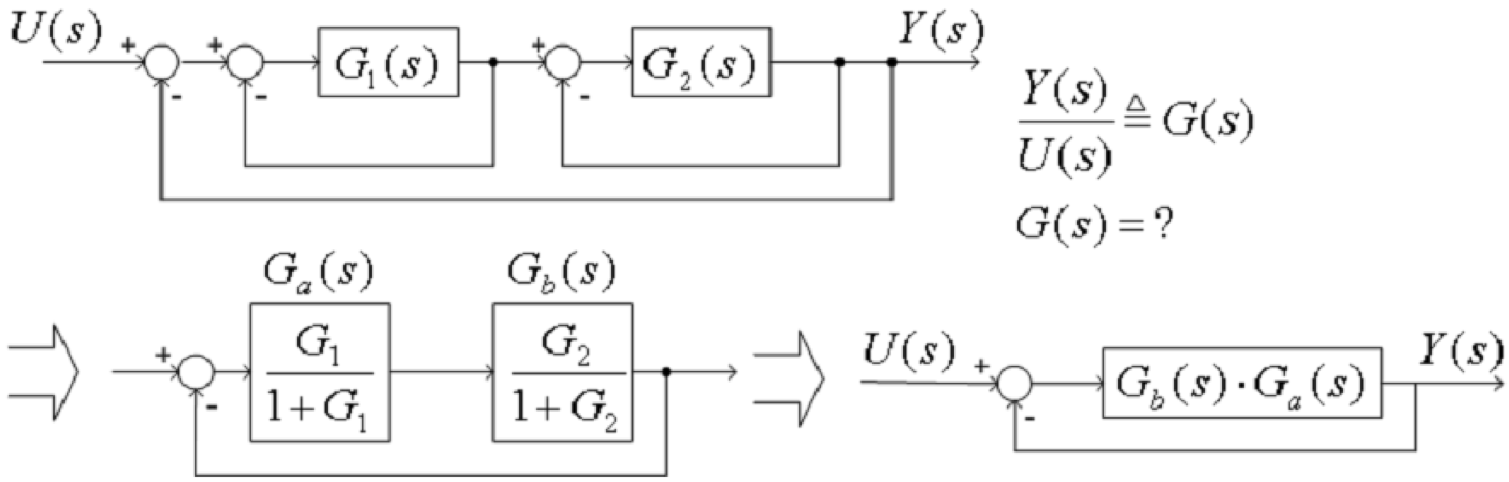
$$M(s) = ?$$

$$Y(s) = M(s)U(s) = \frac{C(s)P(s)}{1 + H(s)C(s)P(s)} U(s)$$

$$\therefore M(s) = \frac{C(s)P(s)}{1 + H(s)C(s)P(s)}$$

Block Diagram

[Example]



$$\therefore G(s)=\frac{G_aG_b}{1+G_aG_b}=\frac{\dfrac{G_1}{1+G_1}\square\dfrac{G_2}{1+G_2}}{1+\dfrac{G_1}{1+G_1}\square\dfrac{G_2}{1+G_2}}=\frac{G_1G_2}{(1+G_1)(1+G_2)+G_1G_2}$$

Signal Flow Graph

□ Signal Flow Graph

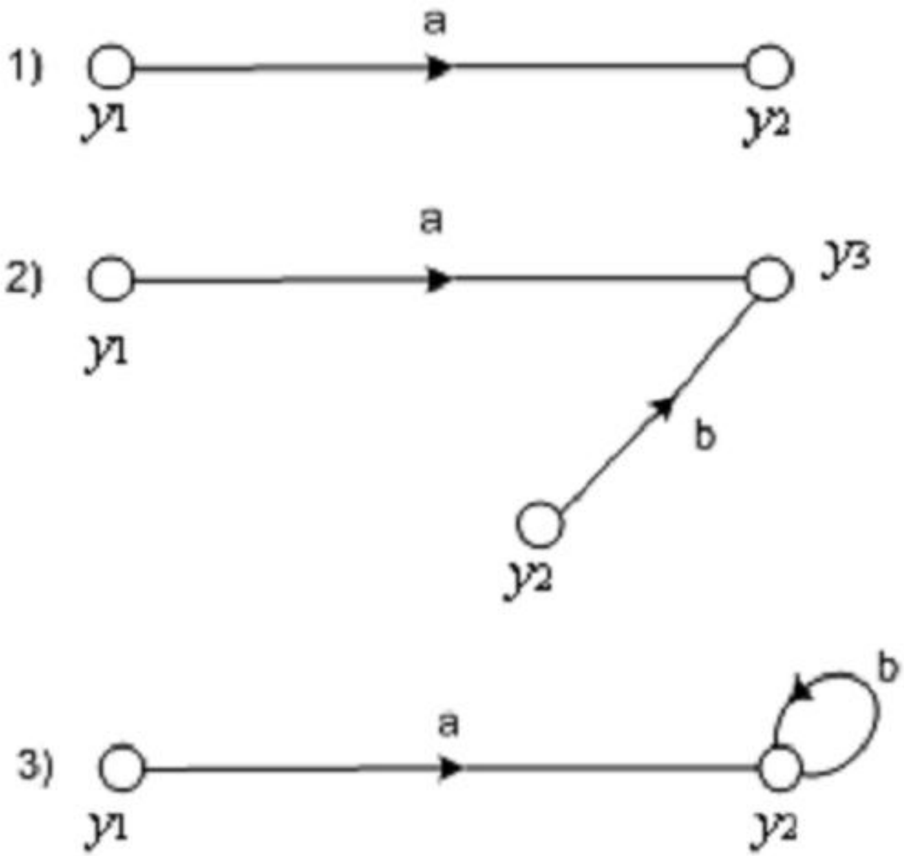
- Alternative approach to block diagram

o Basic elements

· variable → beneath node

· gain → above the directional branch

Signal Flow Graph



1) $y_2 = ay_1$

2) $y_3 = ay_1 + by_2$

3) $y_2 = ay_1 + by_2$

Signal Flow Graph

[Example]

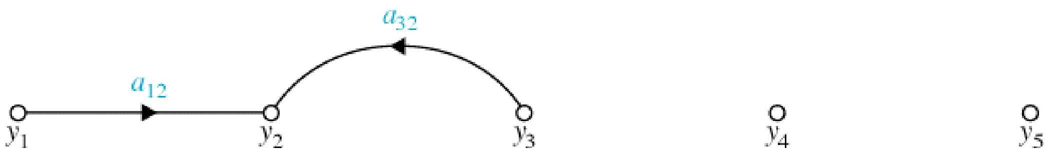
$$y_2 = a_{12}y_1 + a_{32}y_3$$

$$y_3 = a_{23}y_2 + a_{43}y_4$$

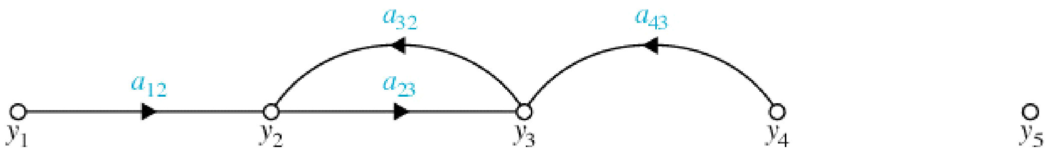
$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$

$$y_5 = a_{25}y_2 + a_{45}y_4$$

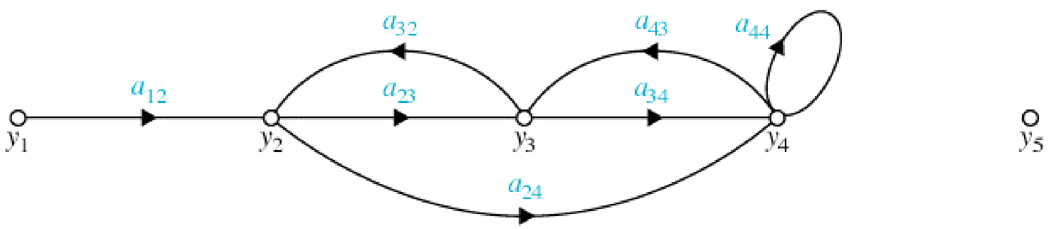
Signal Flow Graph



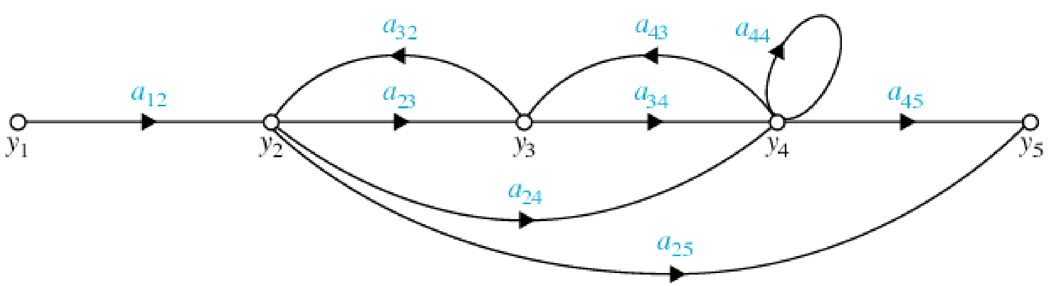
(a) $y_2 = a_{12}y_1 + a_{32}y_3$



(b) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4$



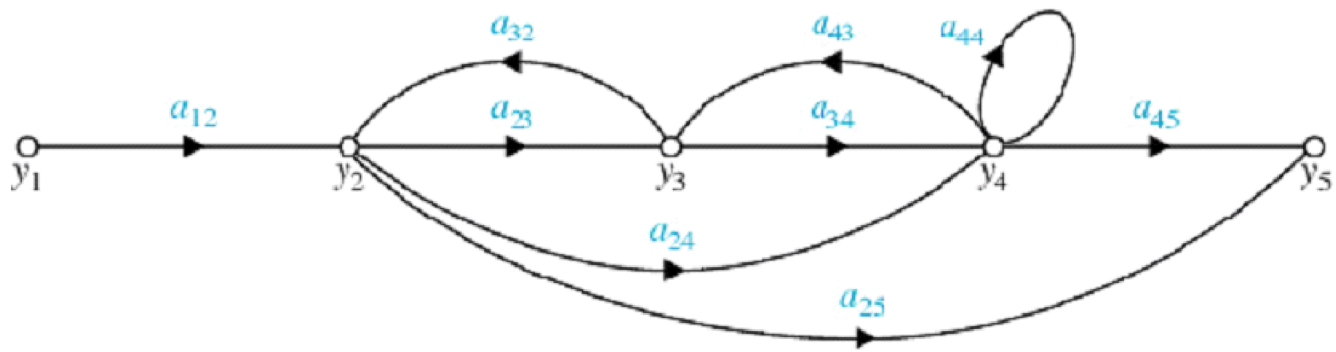
(c) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4 \quad y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$



(d) Complete signal-flow graph

Signal Flow Graph

Definitions of SFG terms



- Input node ; node that has only outgoing branches

  ex) y_1

- Output node ; node that has only incoming branches

  ex) y_5

→ **Can be assigned arbitrarily by using a unit gain branch**

Signal Flow Graph

- Path : collection of continuous branches.
- Path Gain : product of branch gains in the path.
- Forward Path : path that starts at one input node and ends at one output node.

No node is traversed more than once.

- Forward Path Gain : path gain of the forward path.

ex) F. path 1 : $y_1 \to y_2 \to y_3 \to y_4 \to y_5 \Rightarrow M_1 = a_{12}a_{23}a_{34}a_{45}$

path 2 : $y_1 \to y_2 \to y_5 \Rightarrow M_2 = a_{12}a_{25}$

path 3 : $y_1 \to y_2 \to y_4 \Rightarrow M_3 = a_{12}a_{24}a_{45}$

- Loop : a closed path. No node is traversed more than once.
- Loop Gain : product of branch gains in the loop.

Signal Flow Graph

ex) $loop\,1: a_{23} - a_{32}$ $\qquad$ $loop\ gain = a_{23}a_{32}$

$loop\ 2: a_{34} - a_{43}$ $\qquad$ $loop\ gain = a_{34}a_{43}$

$loop\ 3: a_{44}$ $\qquad$ $loop\ gain = a_{44}$

$loop\ 4: a_{24} - a_{43} - a_{32}$ $\qquad$ $loop\ gain = a_{24}a_{43}a_{32}$

- Nontouching Loops : loops that have no common nodes.

ex)

$loop\ \ a_{23} - a_{32}$ and $loop\ \ a_{44}$

Gain Formula for SFG (=Mason's Theorem)

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

N = # of forwards (between y_{in} and y_{out})

Δ = 1 - ($\sum$ single loop gain)

+ ($\sum$ two nontouching loop gain)

- ($\sum$ three nontouching loop gain) + $\cdots$

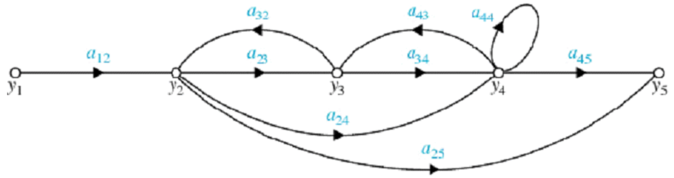
M_k = k - th forward path gain

Δ_k = Δ with k-th forward path removed

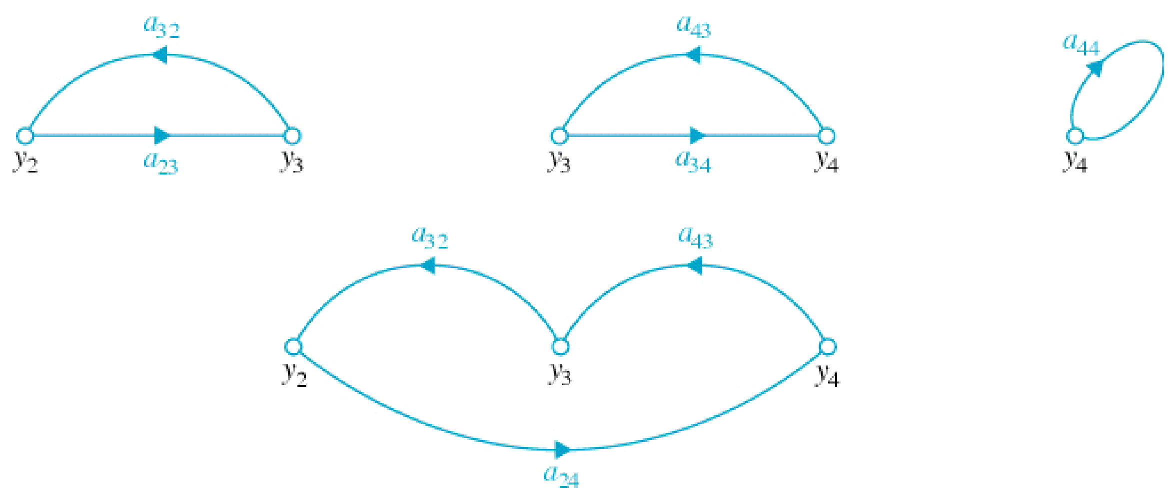
Signal Flow Graph

[Example] Find the gain $\dfrac{y_5}{y_1}$ in the previous graph.

1) Loop gains & Δ



Single loop:



$$L_{11} = a_{23}a_{32}, \quad L_{21} = a_{34}a_{43}, \quad L_{31} = a_{24}a_{43}a_{32}, \quad L_{41} = a_{44}$$

(the 2nd subscript 1 means single loop)

Signal Flow Graph

Two nontouching loop ; $a_{23}a_{32}$ and $a_{44} \rightarrow L_{22} = a_{23}a_{32}a_{44}$

$$\therefore \Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + (L_{22}) = \cdots$$

2) Forward path gain

$$1:\ M_1 = a_{12}a_{23}a_{34}a_{45}$$

$$2:\ M_2 = a_{12}a_{25}$$

$$3:\ M_3 = a_{12}a_{24}a_{45}$$

3) Δ_1 : forward path 1 (X) $\rightarrow$ no loop $\rightarrow \Delta_1 = 1$

Δ_2 : forward path 2 (X) $\rightarrow$ two single loops $a_{34}a_{43}, a_{44}$

$$\therefore \Delta_2 = 1 - (a_{34}a_{43} + a_{44})$$

Signal Flow Graph

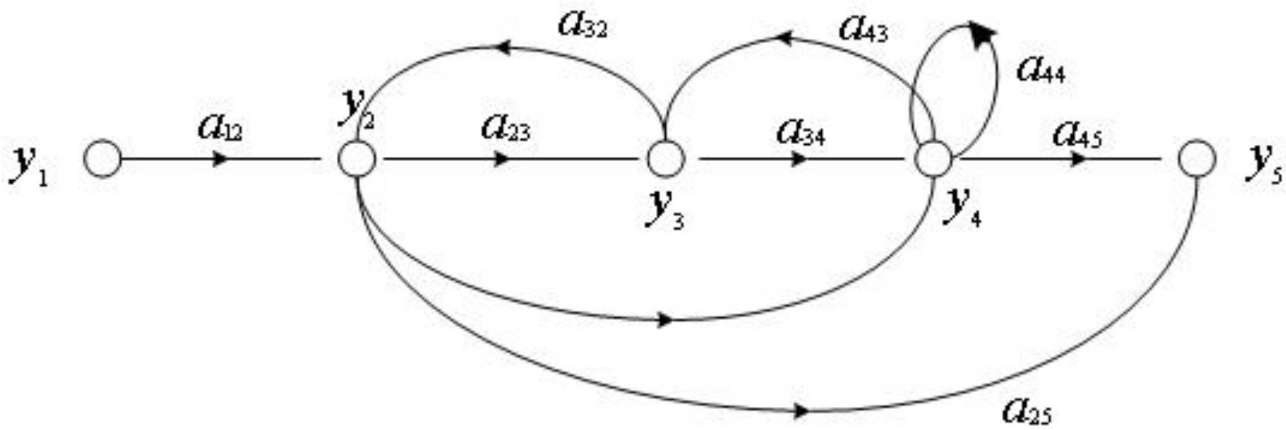
Δ_3 : forward path 3 (X) $\rightarrow$ no loops

$$\Delta_3 = 1$$

$$\therefore \frac{y_5}{y_1} = \frac{\sum M_k \Delta_k}{\Delta} = \cdots$$

Signal Flow Graph

[Example] Find the gain $\dfrac{y_2}{y_1}$: in the previous example.



Signal Flow Graph

1) Loop gains & Δ ; the same

2) forward path 1 : $a_{12} \rightarrow M = a_{12}$

3) Δ_1: forward path 1 removed $\rightarrow$ two single loops

$a_{34}a_{43}$ and a_{44}

$$\therefore \Delta_1 = 1 - (a_{34}a_{43} + a_{44})$$

$$\therefore \frac{y_2}{y_1} = \frac{M_1\Delta_1}{\Delta} = \frac{a_{12}(1 - a_{34}a_{43} - a_{44})}{\Delta}$$