Thermodynamics

Week 12. Entropy Change

Tds Equations

충북대학교
CHUNGBUK NATIONAL UNIVERSITY

GENESYS Laboratory

Objectives

1. Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases
2. Examine a special class of idealized processes, called isentropic processes, and develop the property relations for these processes

The Tds Relations

The differential form of the conservation of energy equation for a closed stationary system (a fixed mass) containing a simple compressible substance

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = TdS$$

$$\delta W_{\text{int rev,out}} = PdV$$

The 1st $T\,ds$, or Gibbs, equation

$$TdS = dU + PdV \qquad \text{(kJ)}$$

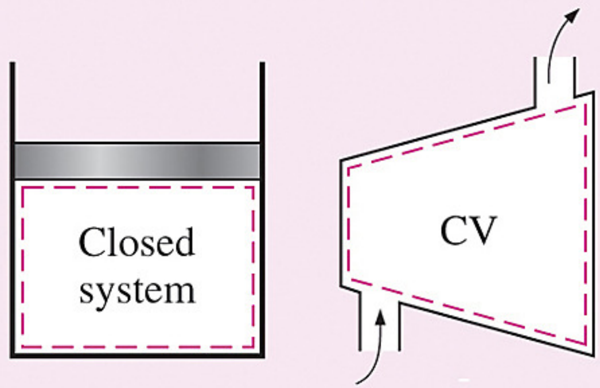
$$Tds = du + Pdv \qquad \text{(kJ/kg)}$$

The 2nd $T\,ds$ equation

$$h = u + Pv$$

$$dh = du + Pdv + vdP$$

$$Tds = dh - vdP$$



$$T\,ds = du + P\,dv$$
$$T\,ds = dh - v\,dP$$

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$
$$= \frac{dh}{T} - \frac{vdP}{T}$$

Entropy Change of Liquids and Solids

Liquids and solids can be approximated as incompressible substances

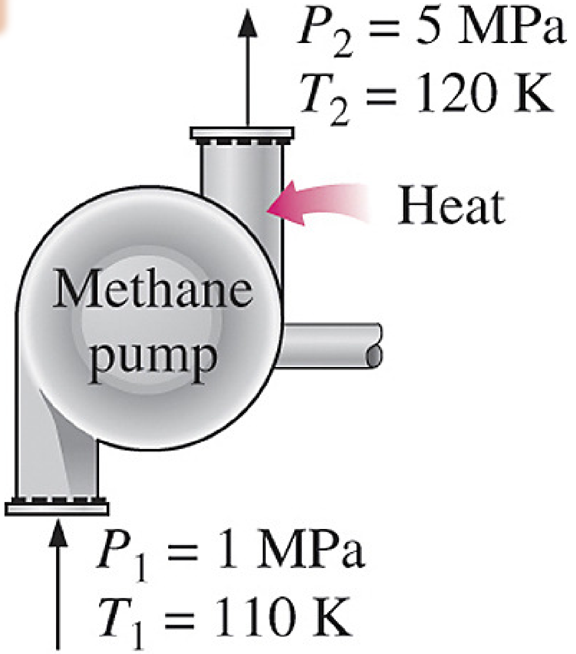
$$ds = \frac{du}{T} + \frac{Pdv}{T} = \frac{cdT}{T} \quad \left(\because dv \cong 0, c_v = c_p = c\right)$$

$$s_2 - s_1 = \int_1^2 \frac{cdT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1} \qquad (\text{kJ/kg} \cdot \text{K})$$

Isentropic case:

$$s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \Rightarrow T_2 = T_1$$

EX 1) Effect of Density of a Liquid on Entropy



EX 2) Economics of Replacing a Valve by a Turbine

1^{st} VS 2^{nd} Laws of Thermodynamics

The first Laws

- Energy conservation
- Quantity point of view

  \- In terms of "Energy"
- Energy cannot be created or destroyed, but it always conserves

  \- If not, it violates 1^{st} law of thermodynamics
- Energy input – Energy output = Energy stored
- Hot → Cold (O); Cold → Hot (X)
- 100% output w/o any loss

The second Laws

- Direction of a process
- Quality point of view

  \- In terms of "Entropy"
- Entropy generation always increases

  \- If not, it violates 2^{nd} law of thermodynamics
- A process increase Entropy high → Irreversibility high
- A process increase Entropy low → Irreversibility low

Entropy Change of Ideal Gases

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$= \frac{dh}{T} - \frac{vdP}{T}$$

$$Pv = RT$$

$$du = c_v dT$$

$$dh = c_p dT$$

The differential entropy change of an ideal gas

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$= c_p \frac{dT}{T} - R\frac{dP}{P}$$

The entropy change for a process obtained by integrating

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

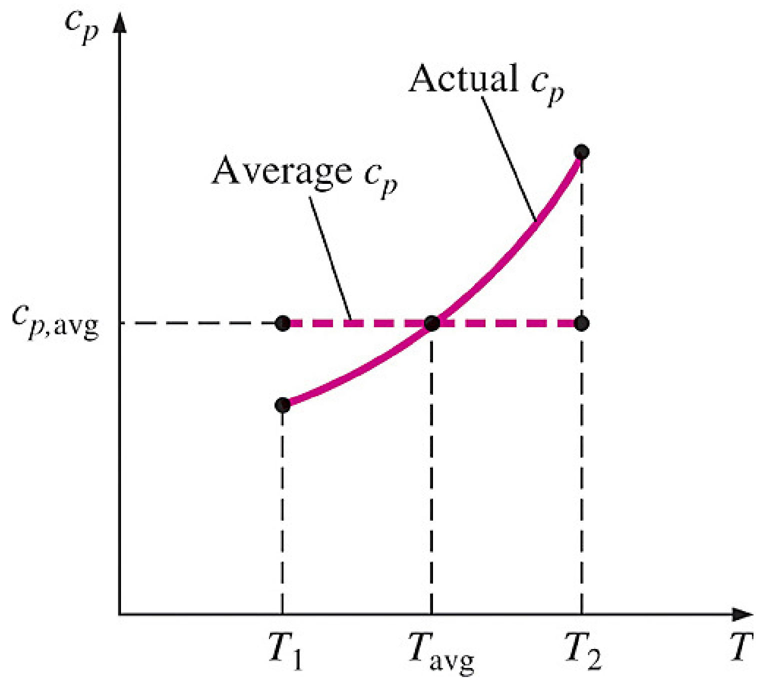
$$= \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

Constant Specific Heats (Approximate Analysis)

The entropy change relations for ideal gases under the constant specific heat assumption

$$s_2 - s_1 = c_{v,\text{avg}} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$= c_{p,\text{avg}} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad (\text{kJ/kg}\cdot\text{K})$$



Entropy changes can also be expressed on a unit mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,\text{avg}} \ln\frac{T_2}{T_1} + R_u \ln\frac{v_2}{v_1}$$

$$= \bar{c}_{p,\text{avg}} \ln\frac{T_2}{T_1} - R_u \ln\frac{P_2}{P_1} \quad (\text{kJ/kmol}\cdot\text{K})$$

Variable Specific Heats (Exact Analysis)

$$s^\circ = \int_0^T c_p(T)\frac{dT}{T}$$

$$\int_1^2 c_p(T)\frac{dT}{T} = s_2^\circ - s_1^\circ$$

T, K	s°, kJ/kg • K
⋮	⋮
300	1.70203
310	1.73498
320	1.76690
⋮	⋮

(Table A-17)

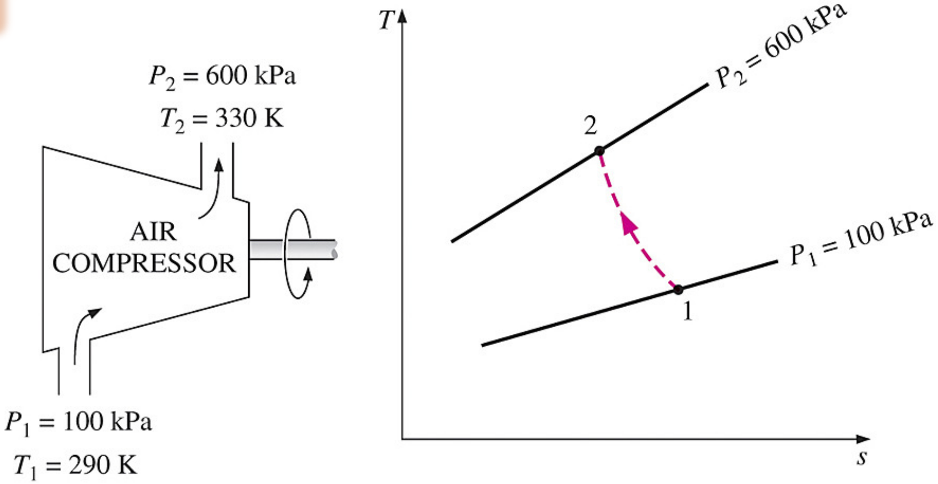
The entropy change relations for ideal gases under the variable specific heat assumption

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1} \quad (\mathrm{kJ/kg\cdot K})$$

It is expressed on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u\ln\frac{P_2}{P_1} \quad (\mathrm{kJ/kmol\cdot K})$$

EX 3) Entropy Change of an Ideal Gas



AIR

$T_1 = 290\ \text{K}$

$T_2 = 330\ \text{K}$

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$
$$= -0.3844\ \text{kJ/kg} \cdot \text{K}$$

$$s_2 - s_1 = C_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$
$$= -0.3842\ \text{kJ/kg} \cdot \text{K}$$

Isentropic Processes of Ideal Gases (Approximate Analysis)

$$s_2 - s_1 = \boxed{c_{v,\text{avg}} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}}$$

$$= \boxed{c_{p,\text{avg}} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}}$$

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1} \Rightarrow \ln\frac{T_2}{T_1} = \ln\left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$$R = c_p - c_v,\ k = {c_p}/{c_v} \Rightarrow {R}/{c_v} = k-1$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad \text{(ideal gas)}$$

2nd isentropic relation

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \text{(ideal gas)}$$

1st isentropic relation

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k} \quad \text{(ideal gas)}$$

3rd isentropic relation

Compact forms

$$Tv^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant} \quad \text{(ideal gas)}$$

$$Pv^{k} = \text{constant}$$

Isentropic Processes of Ideal Gases (Exact Analysis I)

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1} = 0$$

$$s_2^\circ = s_1^\circ + R\ln\frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \exp\frac{s_2^\circ - s_1^\circ}{R} = \frac{\exp\left(s_2^\circ/R\right)}{\exp\left(s_1^\circ/R\right)}$$

Relative pressure

$$P_r = \exp(s^\circ/R)$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1}\frac{P_1}{P_2}$$

$$\frac{v_2}{v_1} = \frac{T_2}{T_1}\frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

Relative specific volume

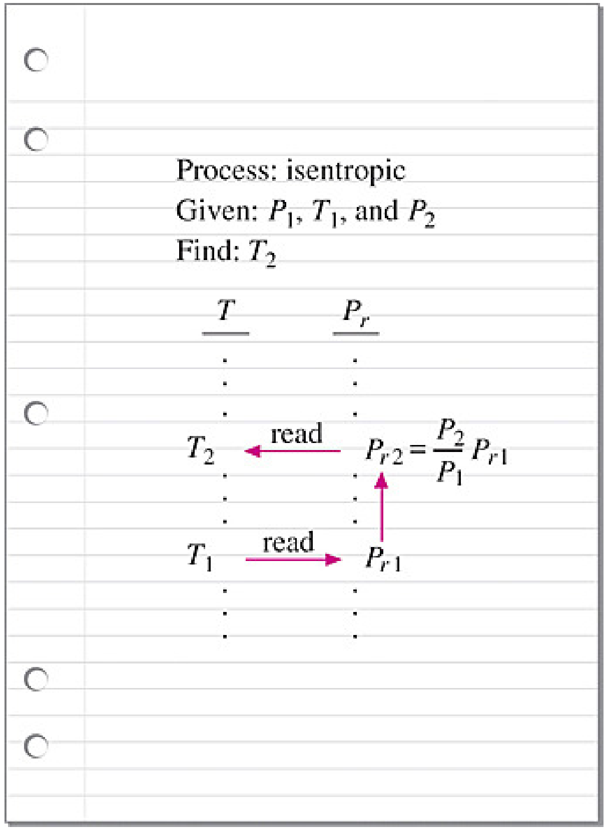
$v_r = T/Pr$

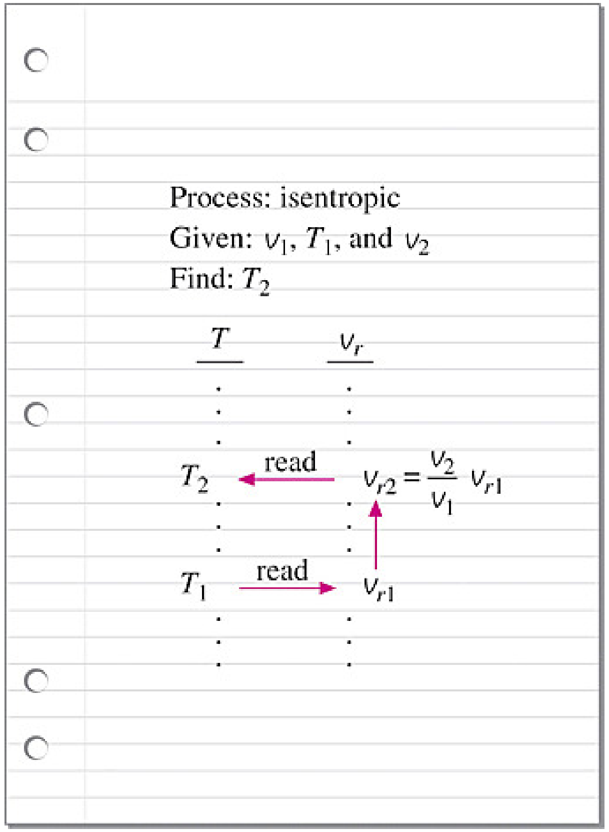
$$\left(\frac{v_2}{v_1}\right)_{s=\text{const}} = \frac{v_{r2}}{v_{r1}}$$

- Strictly valid for isentropic processes of ideal gases only
- The values of P_r and v_r are listed for air in Table A-17

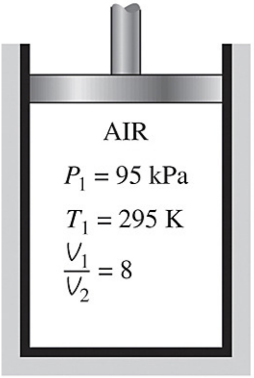
Isentropic Processes of Ideal Gases (Exact Analysis II)

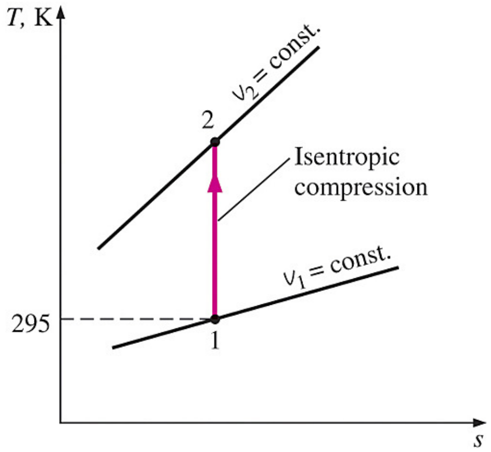
• The values of P_r and v_r listed for air in Table A-17 are used for calculating the final temperature during an isentropic process



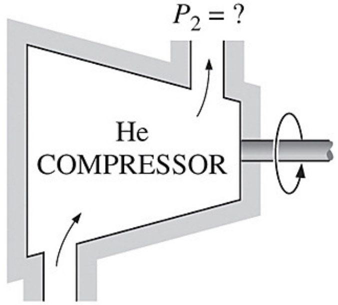


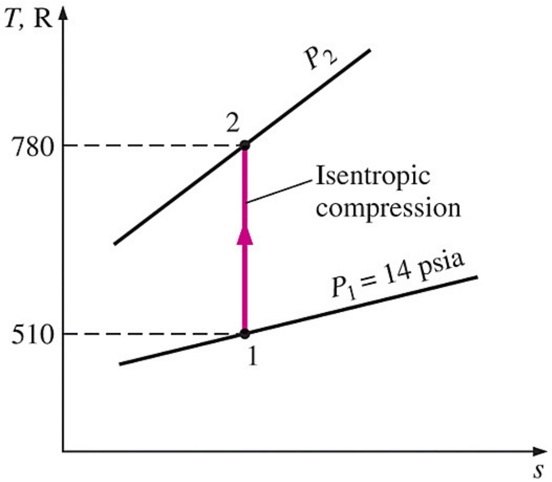
EX 4) Isentropic Compression of Air in a Car Engine





EX 5) Isentropic Compression of an Ideal Gas





Reversible Steady-Flow Work

The energy balance for a steady-flow device undergoing an internally reversible process

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

$$\delta q_{\text{rev}} = Tds \quad \rightarrow \quad -\delta w_{\text{rev}} = vdP + dke + dpe$$

$$Tds = dh - vdP$$

$$w_{\text{rev}} = -\int_1^2 vdP + \Delta ke + \Delta pe \qquad \text{(kJ/kg)}$$

$$w_{\text{rev,in}} = \int_1^2 vdP + \Delta ke + \Delta pe \qquad \text{(kJ/kg)}$$

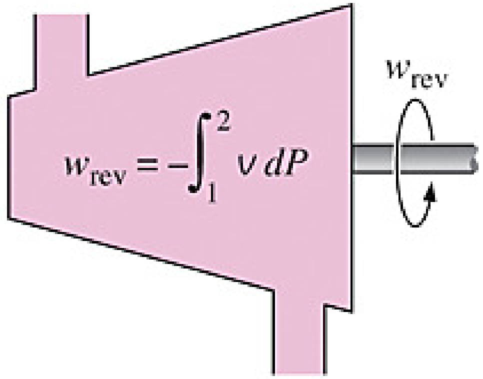
For incompressible fluid

$$w_{\text{rev}} = -v\left(P_2 - P_1\right) - ke - pe \quad \text{(kJ/kg)}$$

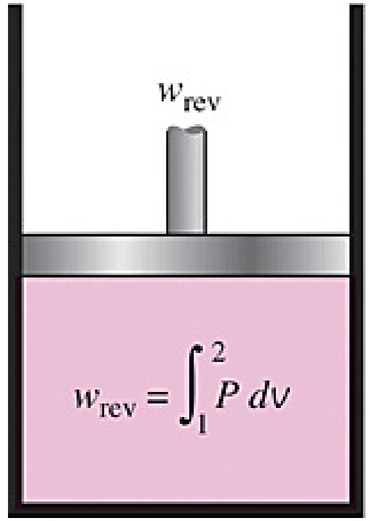
In case of no work interactions,

$$v\left(P_2 - P_1\right) + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right) = 0$$

Bernoulli equation

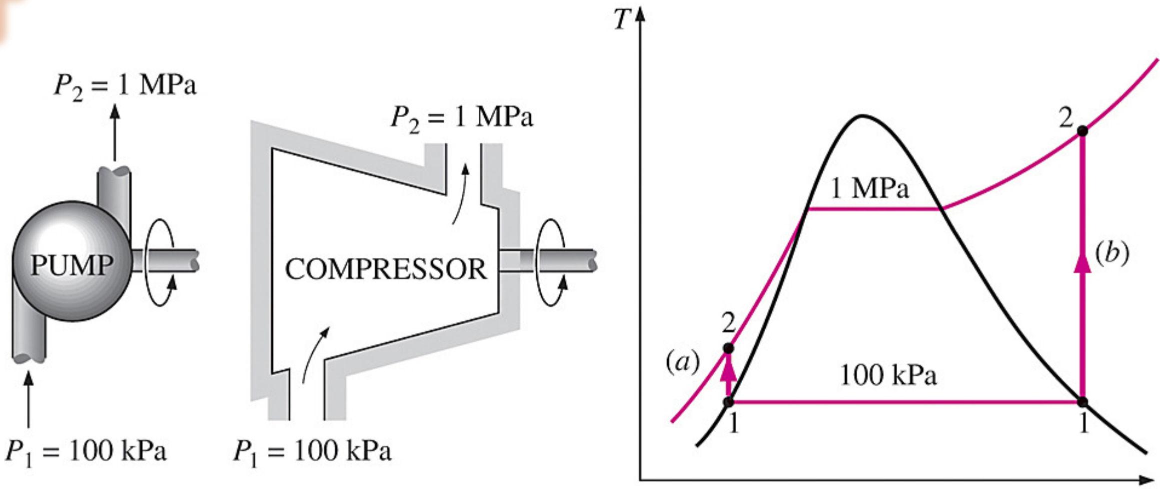


(a) Steady-flow system



(b) Closed system

EX 6) Compressing a Substance in the Liquid versus Gas Phases



(a) Compressing a liquid

(b) Compressing a vapor

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

The energy balance for actual and reversible devices

$$\text{Actual } : \delta q_{\text{act}} - \delta w_{\text{act}} = dh + d\text{ke} + d\text{pe}$$

$$\text{Reversible } : \delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + d\text{ke} + d\text{pe}$$

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

or

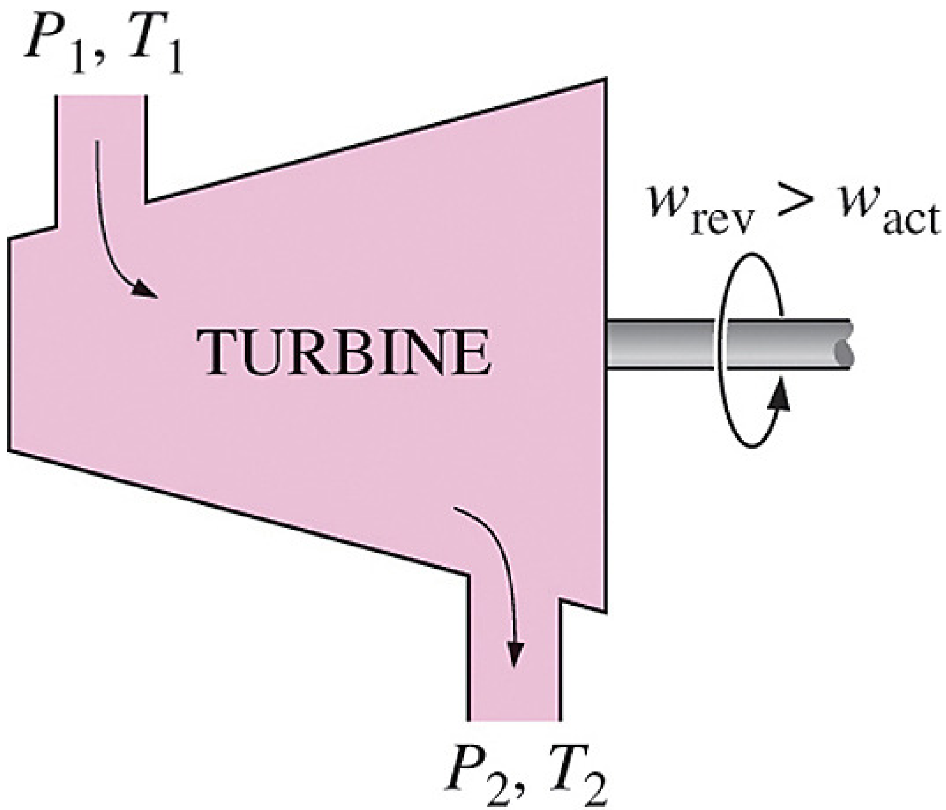
$$\delta q_{\text{rev}} - \delta q_{\text{act}} = \delta w_{\text{rev}} - \delta w_{\text{act}}$$

where, $\delta q_{\text{rev}} = T ds$

$$ds - \frac{\delta q_{\text{act}}}{T} = \frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} \geq 0$$

$$\Rightarrow ds \geq \frac{\delta q_{\text{act}}}{T}$$

$$\Rightarrow w_{\text{rev}} \geq w_{\text{act}} \left(\because T \geq 0 \right)$$



Minimizing The Compressor Work

The compressor work is given by

$$w_{\text{rev,in}} = \int_1^2 v\,dP + \cancel{\Delta ke} + \cancel{\Delta pe} \qquad \text{(kJ/kg)}$$

$$= \int_1^2 v\,dP$$

- Ways of minimizing the compressor work is
  1. Minimizing the irreversibilities such as friction, turbulence, and nonquasi-equilibrium compression
  2. Keeping the specific volume of the gas as small as possible during the compression process, which can be achieved by maintaining the temperature of the gas as low as possible
- Effect of cooling during the compression process

2nd isentropic relation

Isentropic process : $Pv^k = \text{constant}$

$$w = -\int_1^2 \frac{P_1^{1/k} v_1}{P^{1/k}} dP = -P_1^{1/k} v_1 \int_1^2 \frac{dP}{P^{1/k}} = \frac{k}{k-1}\left(P_1 v_1 - P_2 v_2\right) = \frac{kR\left(T_1 - T_2\right)}{k-1} = \frac{kRT_1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k}\right]$$

$$w_{\text{comp,in}} = \frac{kR\left(T_2 - T_1\right)}{k-1} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] = \frac{k}{k-1} P v_1 \left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Minimizing The Compressor Work (Continue)

Polytropic process : $Pv^n = \text{constant}$

$$w = \frac{n}{n-1}\left(P_1 v_1 - P_2 v_2\right) = \frac{nR\left(T_1 - T_2\right)}{n-1} = \frac{nRT_1}{n-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{(n-1)/n}\right]$$

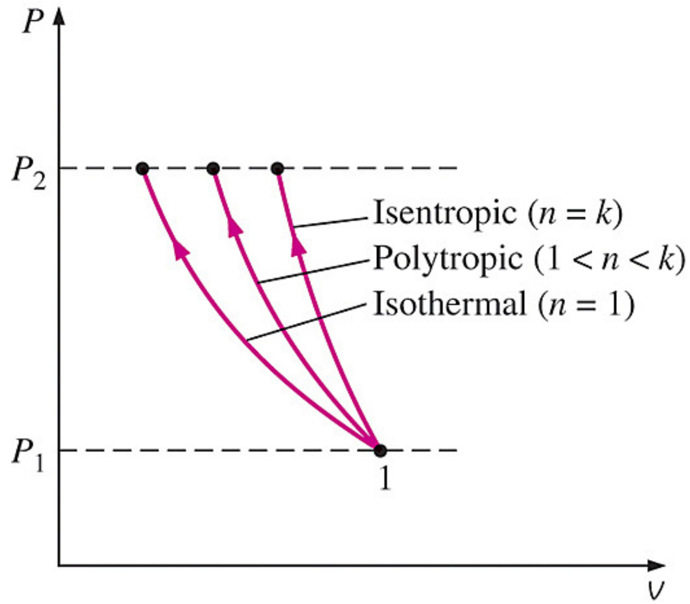
$$w_{\text{comp,in}} = \frac{nR\left(T_1 - T_2\right)}{n-1} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] = \frac{n}{n-1} P v_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Isothermal process : $Pv = \text{constant}$

$$w = -\int_1^2 v\,dP, \quad Pv = P_1 v_1$$

$$= -\int_1^2 P_1 v_1 \frac{dP}{P} = -P_1 v_1 \ln\frac{P_2}{P_1} = -RT \ln\frac{P_1}{P_2}$$

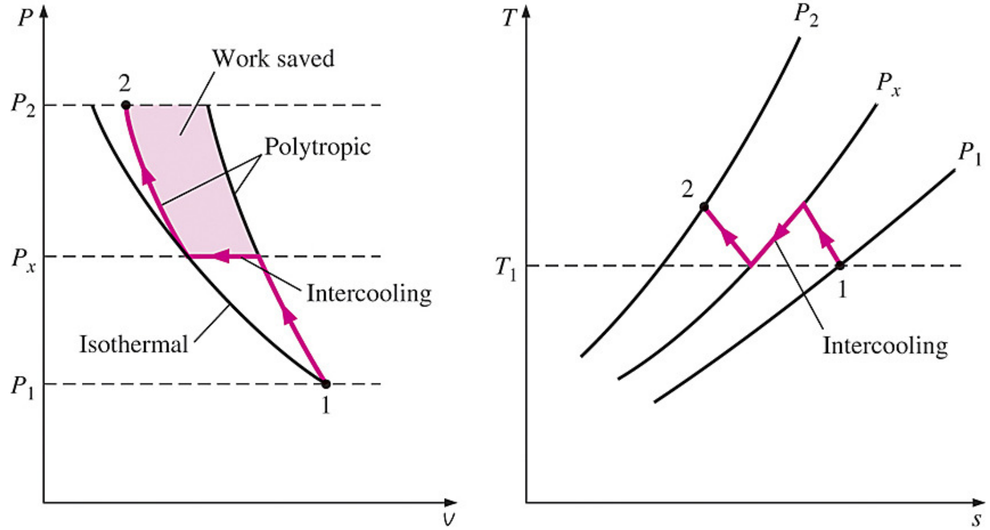
$$w_{\text{comp,in}} = RT \ln\frac{P_2}{P_1}$$



Multistage Compression with Inter-cooling

• The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an inter-cooler

• The effect of intercooling on compressor work is graphically illustrated on *P-v* and *T-s* diagrams



$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}} = \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

P_x value that minimizes the total work $w_{\text{comp,in}}$ is

$$P_x = \left(P_1 P_2\right)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x} \quad \text{if so,} \quad w_{\text{comp I,in}} = w_{\text{comp II,in}}$$

EX 7) Work Input for Various Compression Processes