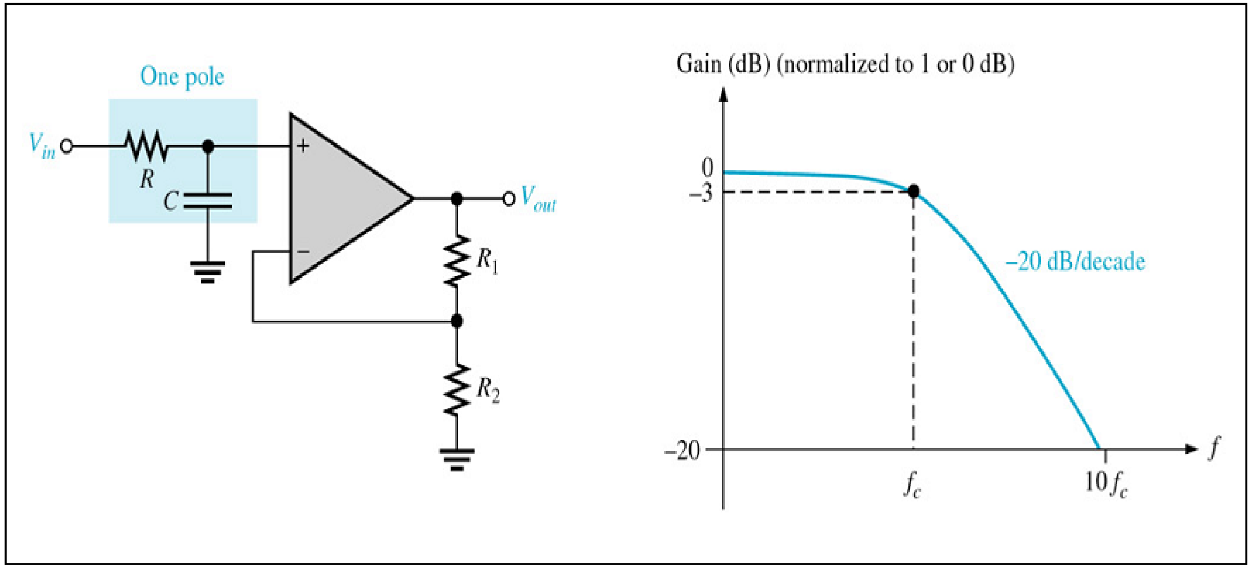
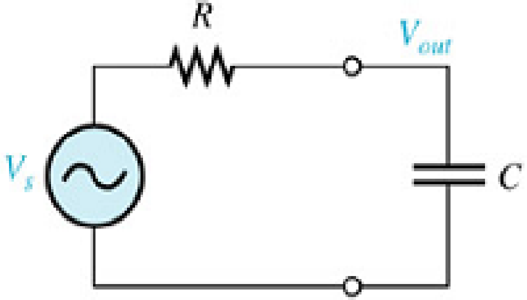
Ch. 15
Active Filters



기본적인 필터 응답(Basic filter response)

Yun SeopYu

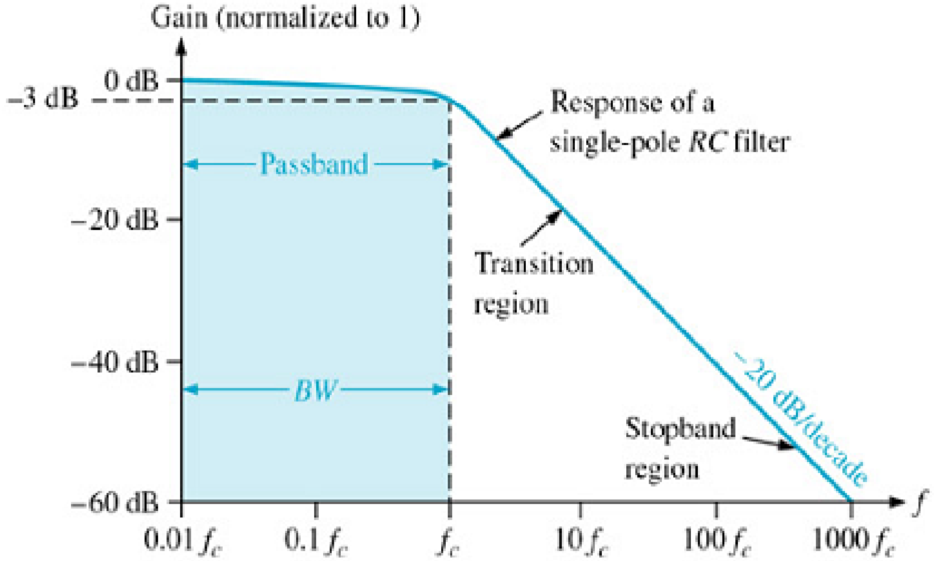
- 저역통과 필터 응답 (low-pass filter (LPF) response)



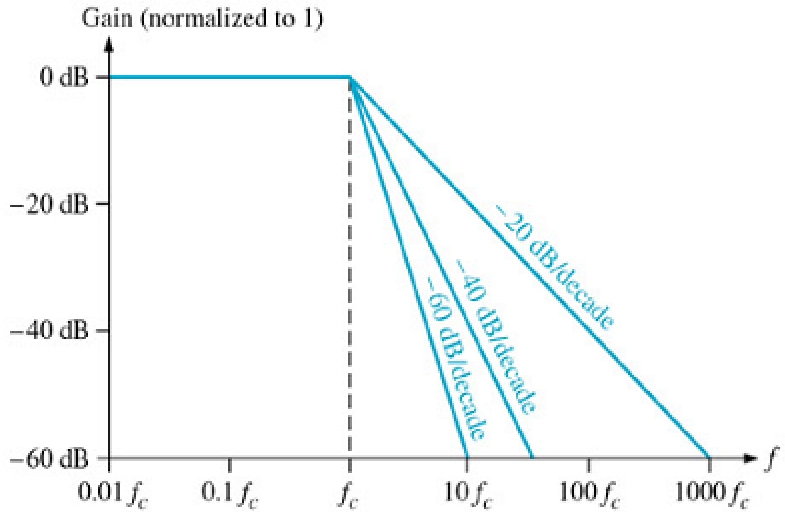
$$A_{v(db)} = 20\log\frac{V_{out}}{V_s} = 20\log\frac{X_C}{\sqrt{R^2+X_C^2}} = 20\log\frac{1}{\sqrt{1+\frac{R^2}{X_C^2}}} = 20\log\frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

$$when \quad R = X_C = \frac{1}{2\pi fC} \text{ --> } f_C = \frac{1}{2\pi RC}$$

Basic LPF response



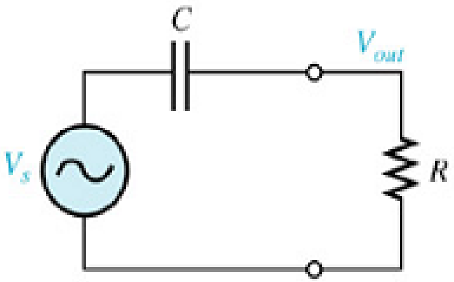
LPF with different roll-off rates



기본적인 필터 응답

Yun SeopYu

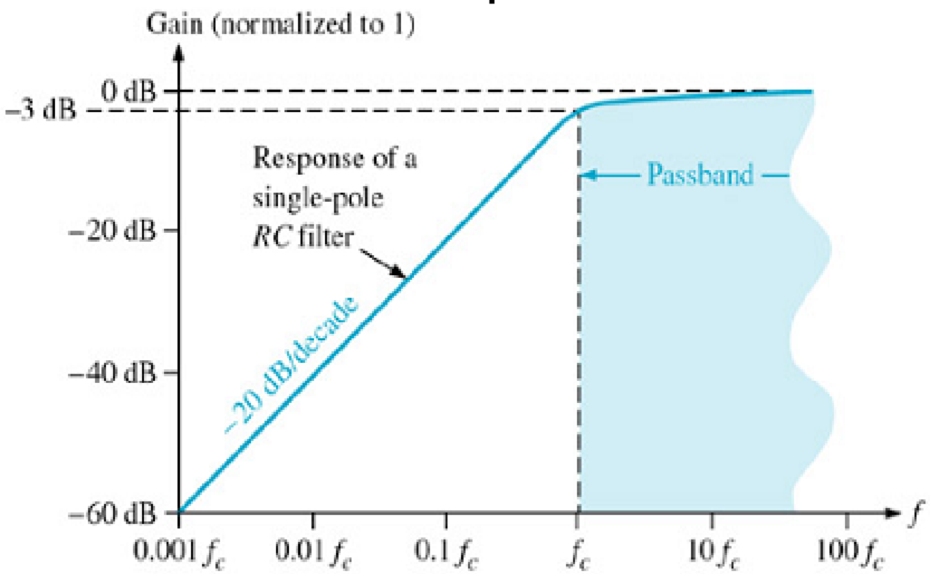
고역통과 필터 응답 (high-pass filter (HPF) response)



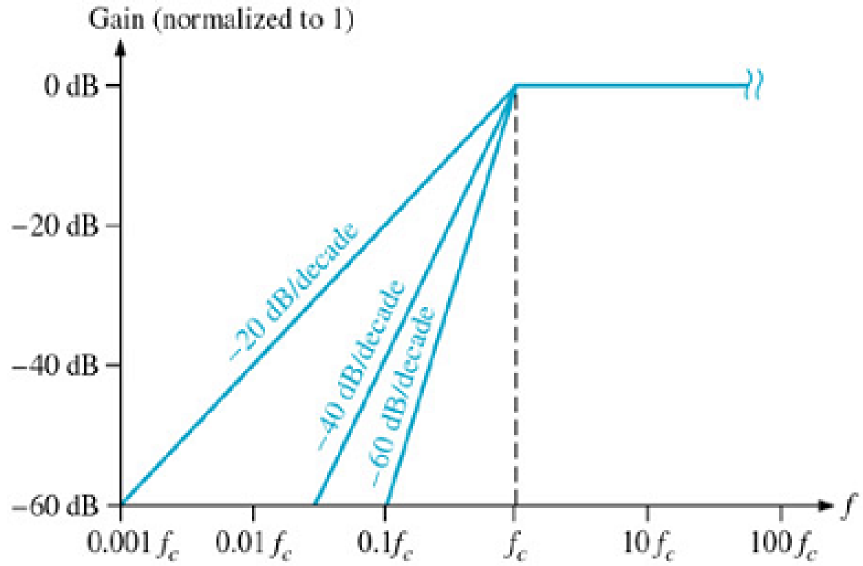
$$A_{v(db)} = 20\log\frac{V_{out}}{V_s} = 20\log\frac{R}{\sqrt{R^2+X_C^2}} = 20\log\frac{1}{\sqrt{1+\frac{X_C^2}{R^2}}} = 20\log\frac{1}{\sqrt{1+\left(\frac{1}{2\pi fRC}\right)^2}}$$

$$when \qquad R = X_C = \frac{1}{2\pi fC} --> f_C = \frac{1}{2\pi RC}$$

<u>Basic HPF response</u>

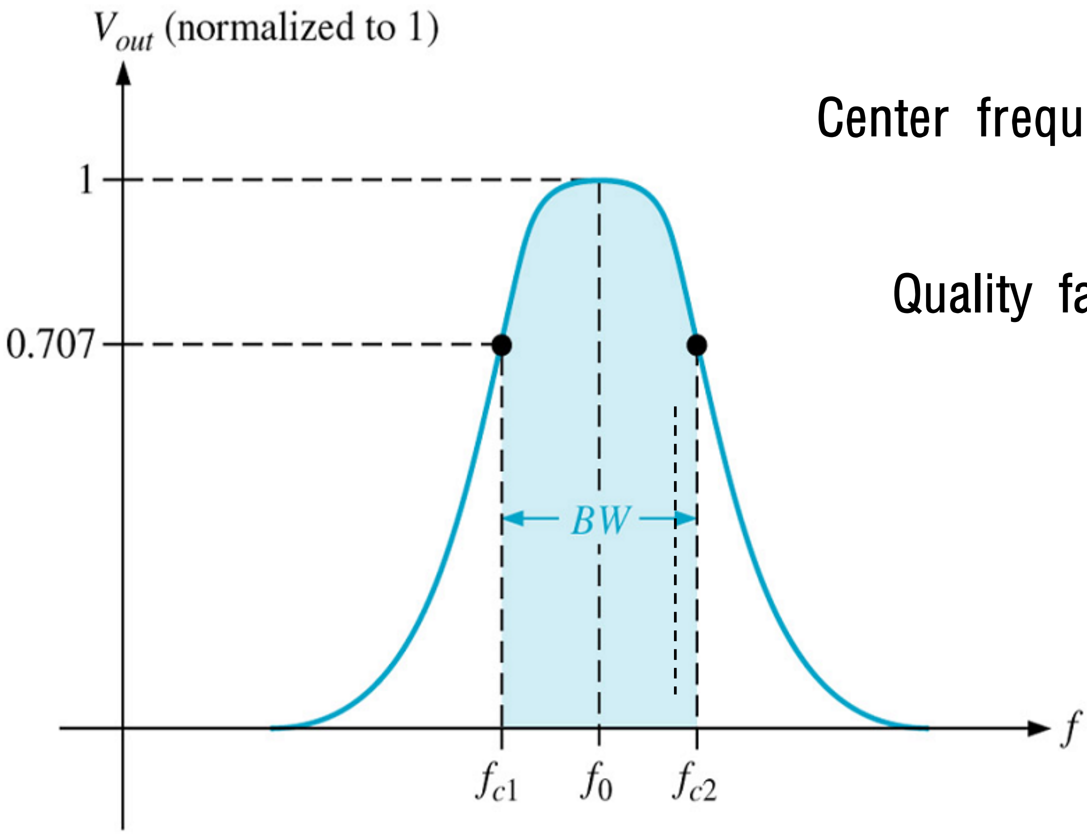


<u>HPF with different roll-off rates</u>



기본적인 필터 응답

- 대역통과 필터 응답 (band-pass filter (BPF) response)



BW = f_{c2} - f_{c1}

Center frequency: $$f_o = \sqrt{f_{c1} f_{c2}}$$

Quality factor: $$Q = \frac{f_o}{BW}$$

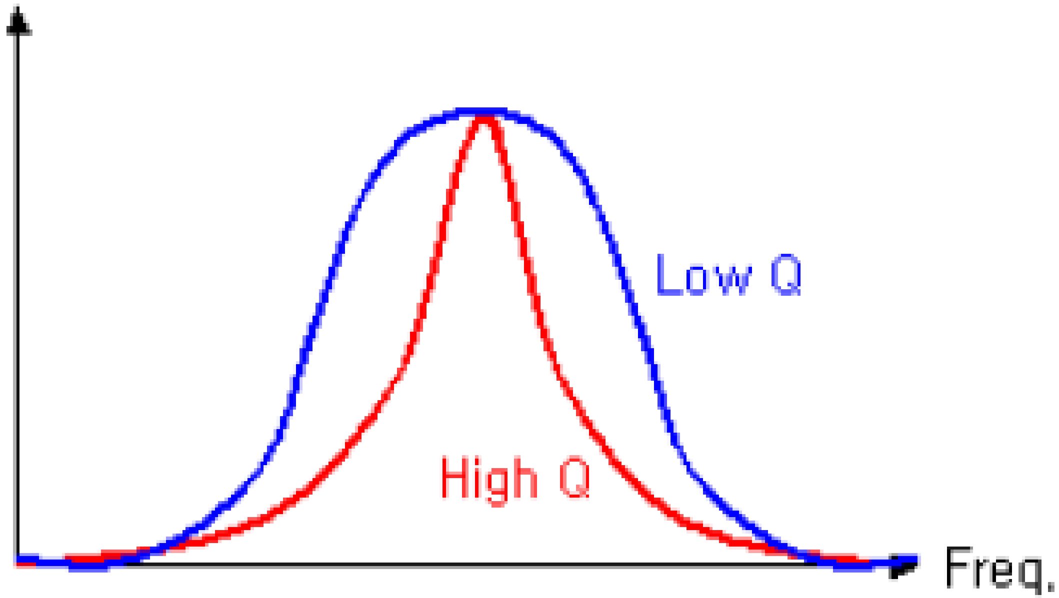
Q is an indication of the selectivity of a BPF.
Narrow BPF: Q > 10.
Wide-band BPF: Q < 10.

Damping Factor: $$DF = {}^{1}\!/_{Q}$$

기본적인 필터 응답

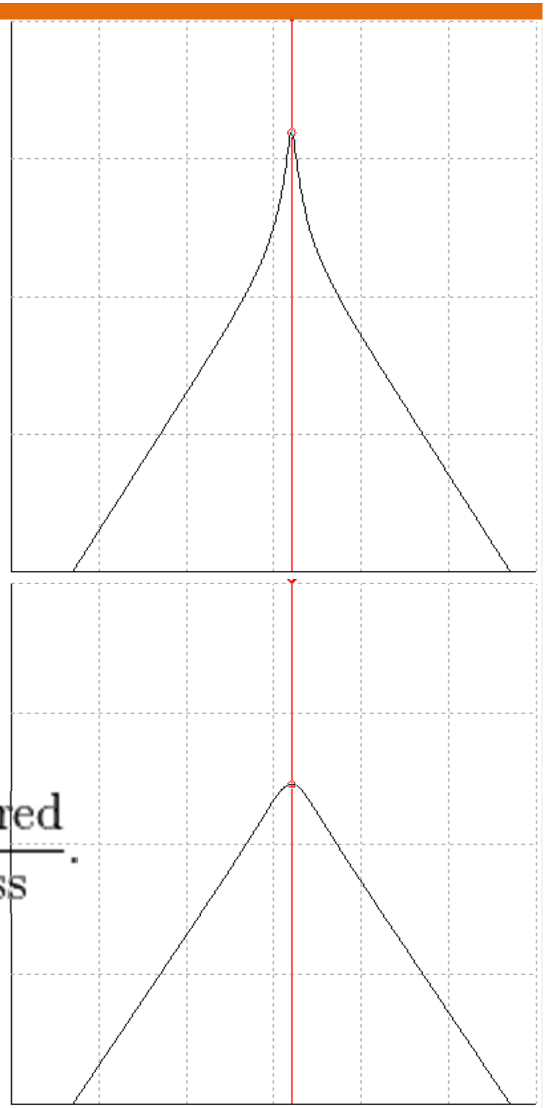
Yun SeopYu

- Q 값: 주파수 선택 특성 품질



$$Q = 2\pi \times \frac{\text{Energy Stored}}{\text{Energy dissipated per cycle}} = 2\pi f_r \times \frac{\text{Energy Stored}}{\text{Power Loss}}.$$

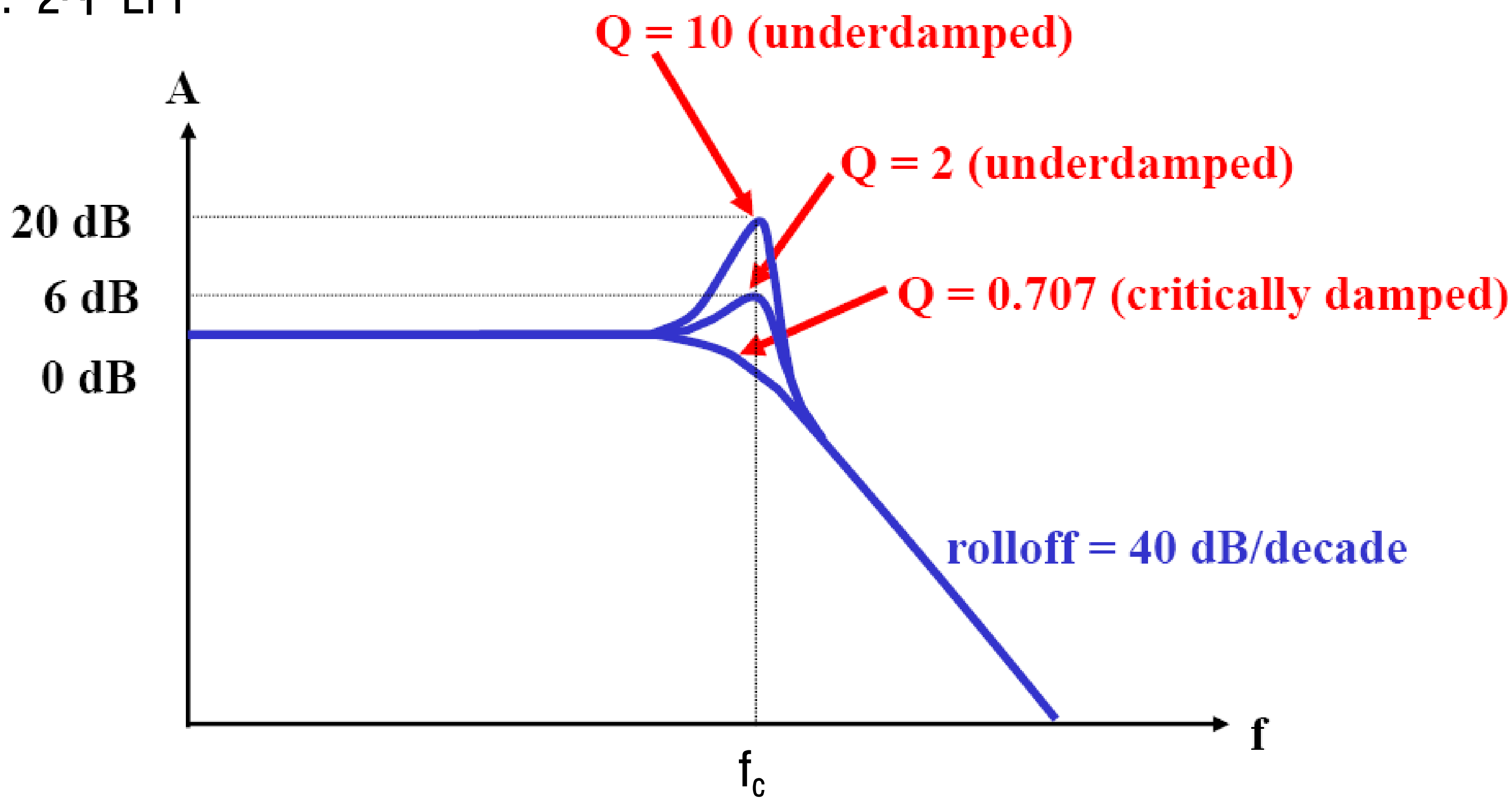
$$Q(\omega) = \omega \times \frac{\text{Maximum Energy Stored}}{\text{Power Loss}},$$



기본적인 필터 응답

- D 값: 댐핑 성질 결정

예: 2차 LPF



$$DF = \frac{1}{Q}$$

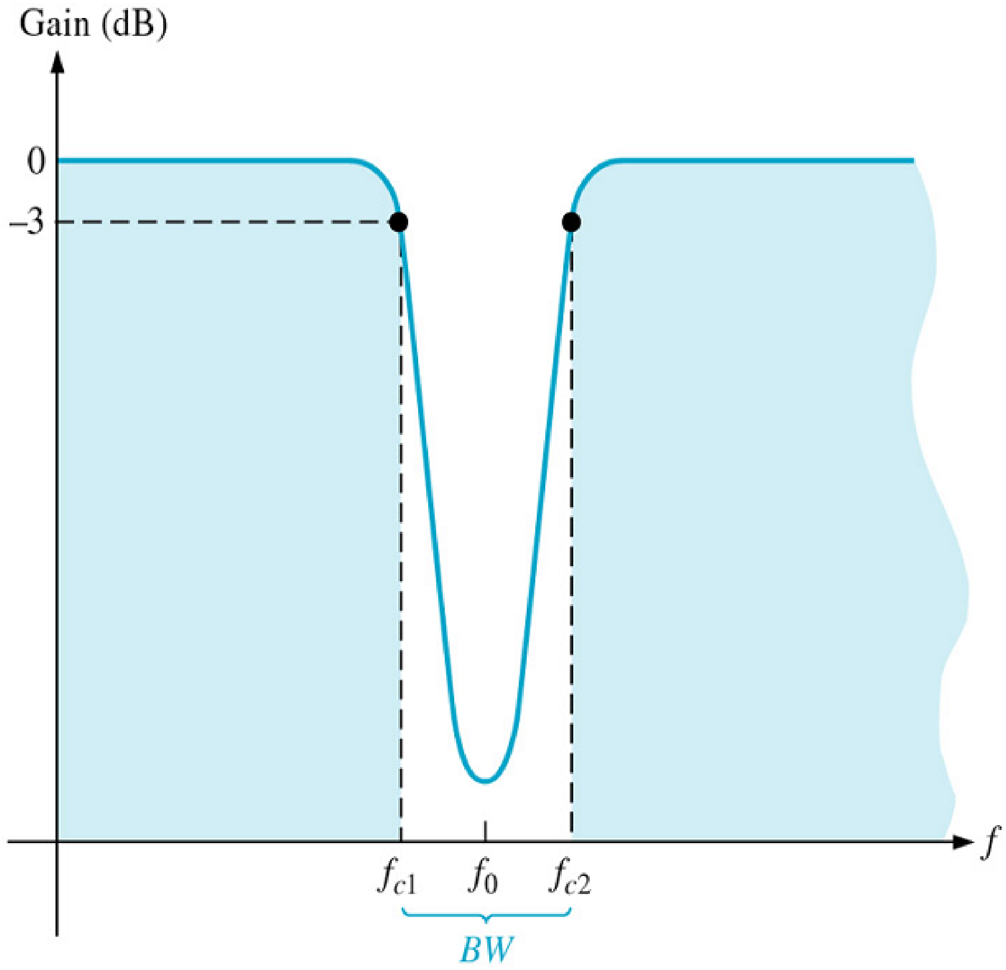
예제 15-1

- f_0 = 15 kHz, BW= 1 kHz, Q ?

$$Q = \frac{f_o}{BW} = \frac{15\text{k}}{1\text{k}} = 15$$ > 10 → 협대역 (narrow BW)

기본적인 필터 응답

대역저지 필터 응답 (band-stop filter response)

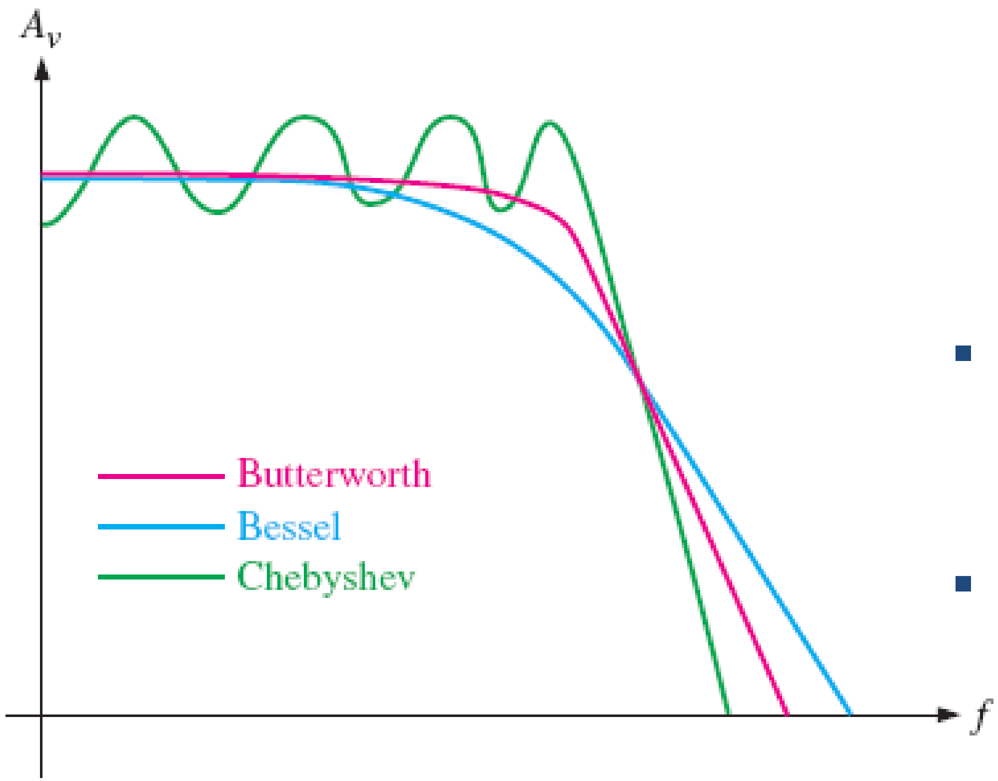


- Band-reject, or notch filter.
- 어떤 특정 주파수 대역만 저지하고 그 외 주파수 대역은 통과시키는 필터

필터 응답 특성(Filter response characteristics)

Yun SeopYu

- 기본 필터 응답 특성



- Butterworth: 통과 대역 매우 평탄
  - roll-off rate: -20 dB/dec/pole
  - 위상지연: 비선형

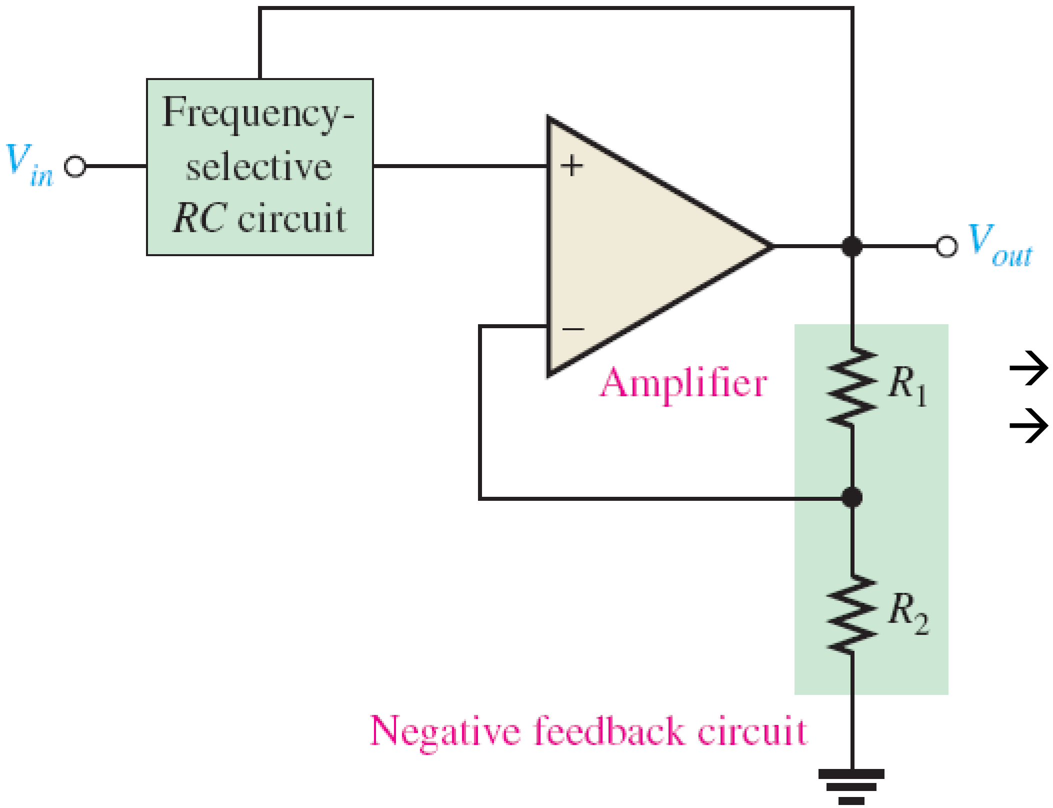
    → 펄스의 상승, 하강 edge의 주파수 부품들의 서로 다른 위상지연 → 출력의 overshoot
  - 통과 대역의 모든 주파수의 이득이 모두 같아야 할 때
- Chebyshev:
  - roll-off rate > -20 dB/dec/pole
  - 통과대역 리플→ 비선형 위상 응답
- Bessel: 선형적인 위상 응답

  → No overshoot 출력
  - roll-off rate < -20 dB/dec/pole.

필터 응답 특성

- 댐핑 계수 (damping factor: DF)

General diagram of active filter



DF: 필터의 응답 특성을 결정

$$DF = 2 - \frac{R_1}{R_2}$$

→ 부귀환 동작으로 필터응답에 영향
→ 필터의 차수 (극점 수)와 관련된 올바른 응답 특성을 구현하기 위해서 댐핑계수 값이 요구
← 표 15-1 참고

표 15-1

Values for the Butterworth response

		1ST STAGE			2ND STAGE			3RD STAGE		
ORDER	ROLL-OFF DB/DECADE	POLES	*DF*	R_1/R_2	POLES	*DF*	R_3/R_4	POLES	*DF*	R_5/R_6
1	−20	1	Optional							
2	−40	2	1.414	0.586						
3	−60	2	1.00	1	1	1.00	1			
4	−80	2	1.848	0.152	2	0.765	1.235			
5	−100	2	1.00	1	2	1.618	0.382	1	0.618	1.382
6	−120	2	1.932	0.068	2	1.414	0.586	2	0.518	1.482

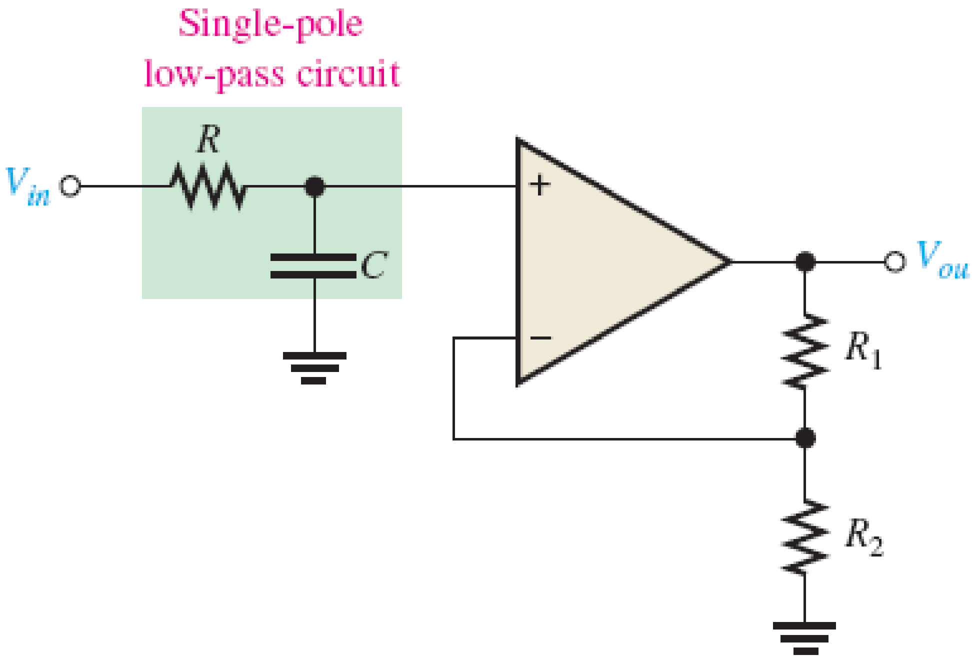
필터 응답 특성

- 댐핑 계수 (damping factor: DF)
  - 극점 (pole): 한 개의 저항과 캐패시터로 구성된 간단한 회로
    - Roll-off: -20dB/decade/pole

- 예제: 2차 버터워스 (Butterworth) 응답
    - 극점: 2개 → DF = 1.414 ← 표15-1
    - $R_1/R_2 = 2 - DF = 2 - 1.414 = 0.586$
    - 비반전 증폭기 전압이득 $A_{cl(NI)} = 1 + R_1/R_2 = 1.586$
    - 만약 $R_2 = 10\ k\Omega$ → $R_1 = 5.86\ k\Omega$

필터 응답 특성

차단 주파수와 기울기

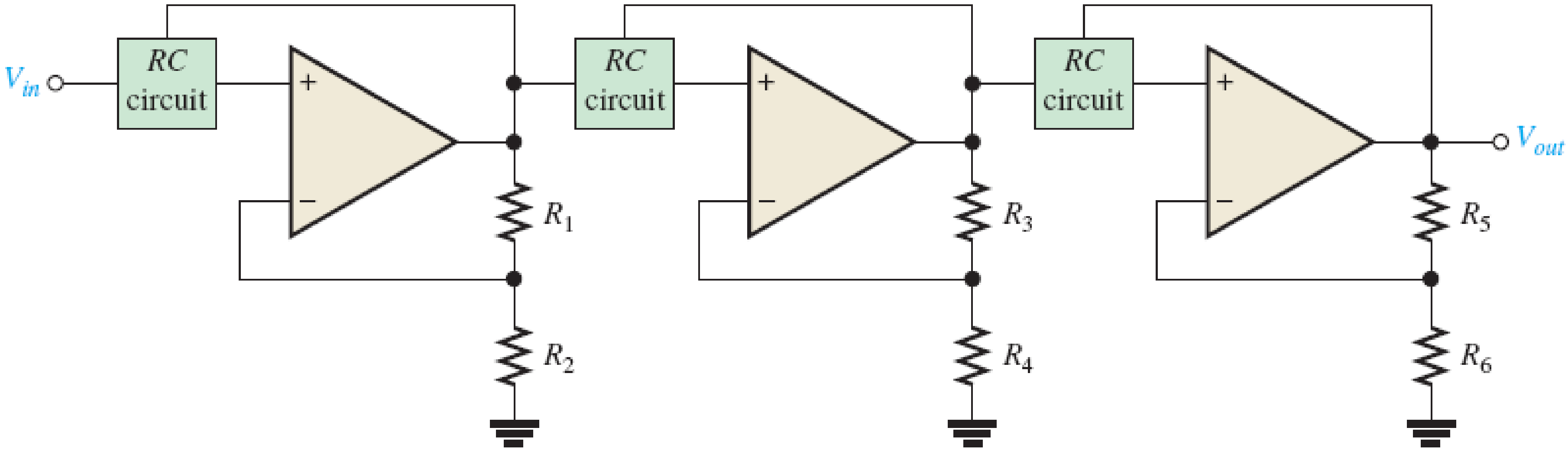


$$f_c = \frac{1}{2\pi RC}$$

Roll-off: -20 dB/dec

필터 응답 특성

차단 주파수와 기울기



-20 dB/dec　　-40 dB/dec　　-60 dB/dec

능동 저역 통과 필터 (Active Low-Pass Filter)

Yun SeopYu

능동 필터의 장점

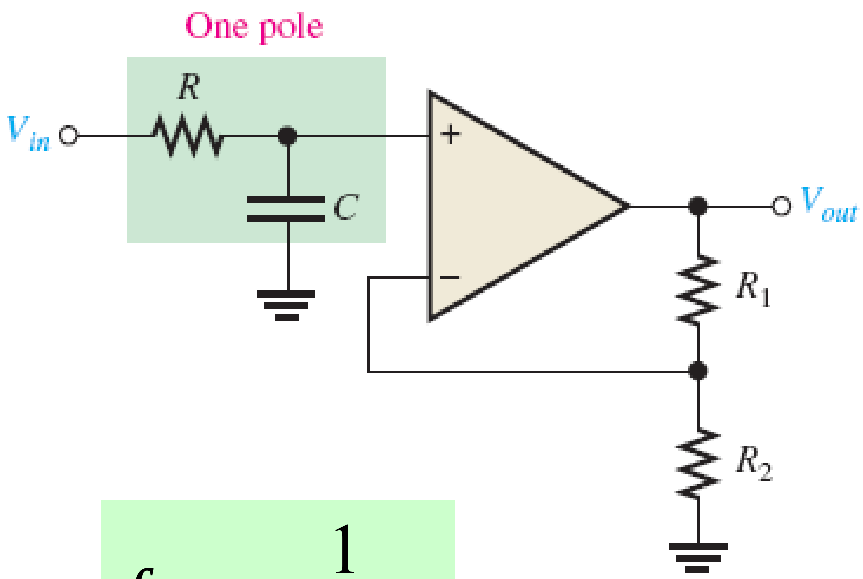
- 연산 증폭기는 필터를 통과에 의한 이득 감쇠를 고려하여 높은 전압 이득 제공
- 높은 입력 임피던스: 구동원의 과부하를 막아줌
- 낮은 출력 임피던스: 필터가 구동될 때 부하의 영향으로부터 필터를 보호
- 요구되는 응답을 수정할 필요 없이 넓은 주파수 범위에서 조정하기 쉬움

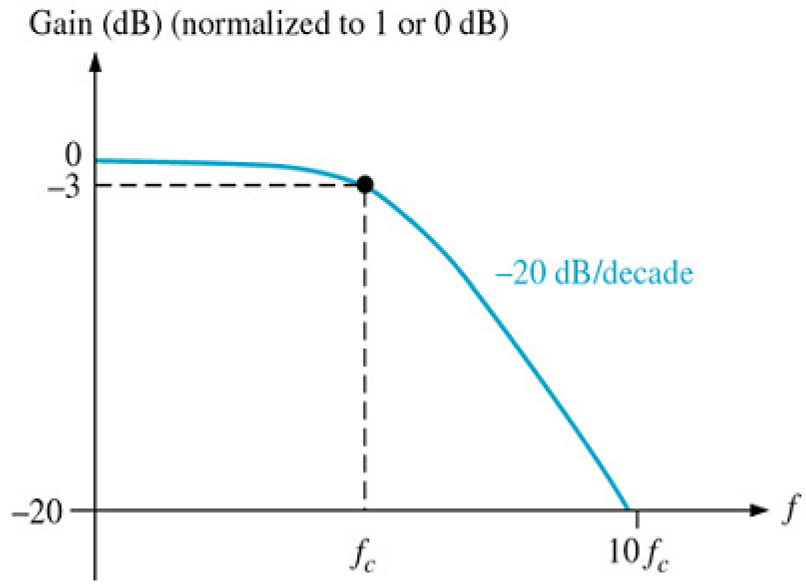
단점

- dc 전원 필요
- 연산증폭기의 주파수 응답에 의해서 제한되어질 수 있다.

능동 저역 통과 필터

1차 단극 필터





$$f_c = \frac{1}{2\pi RC}$$

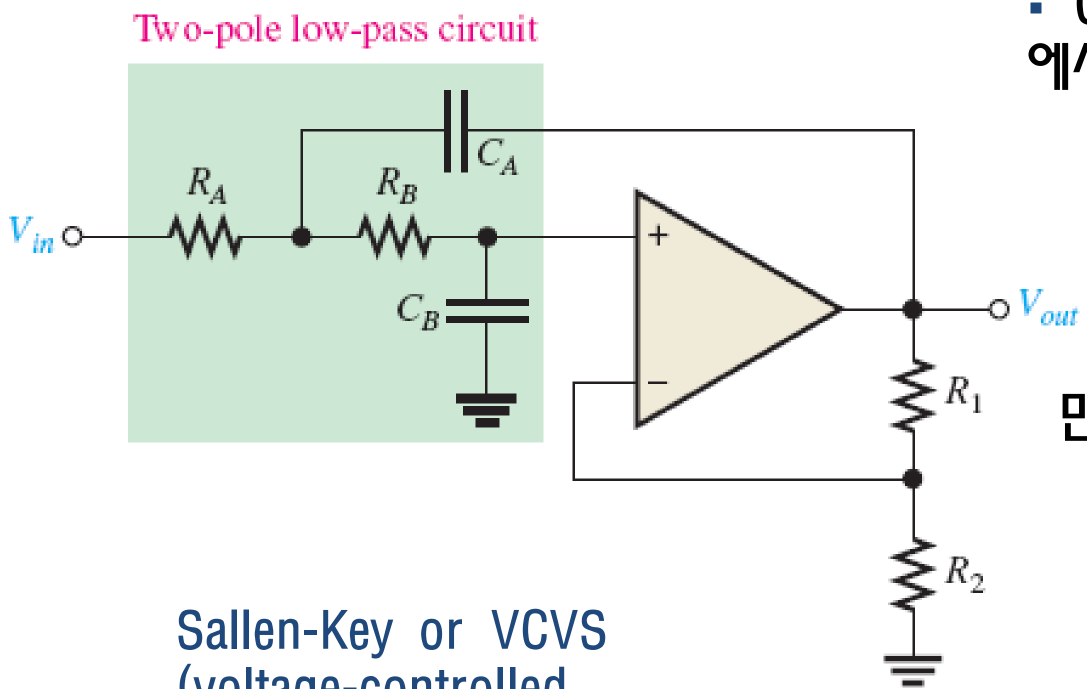
$$A_{cl} = 1 + \frac{R_1}{R_2}$$

Roll-off rate for a single-pole Filter: -20 dB/decade.

A_{cl}: selectable since DF is optional for single-pole LPF

능동 저역 통과 필터

- Sallen-Key 저역통과 필터



Sallen-Key or VCVS (voltage-controlled voltage-source) second-order low-pass filter

- -40dB/decade의 기울기를 가지는 2개의 저역통과 필터
- C_A에 의해서 통과대역 모서리 근처에서 예리한 응답을 갖는다

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

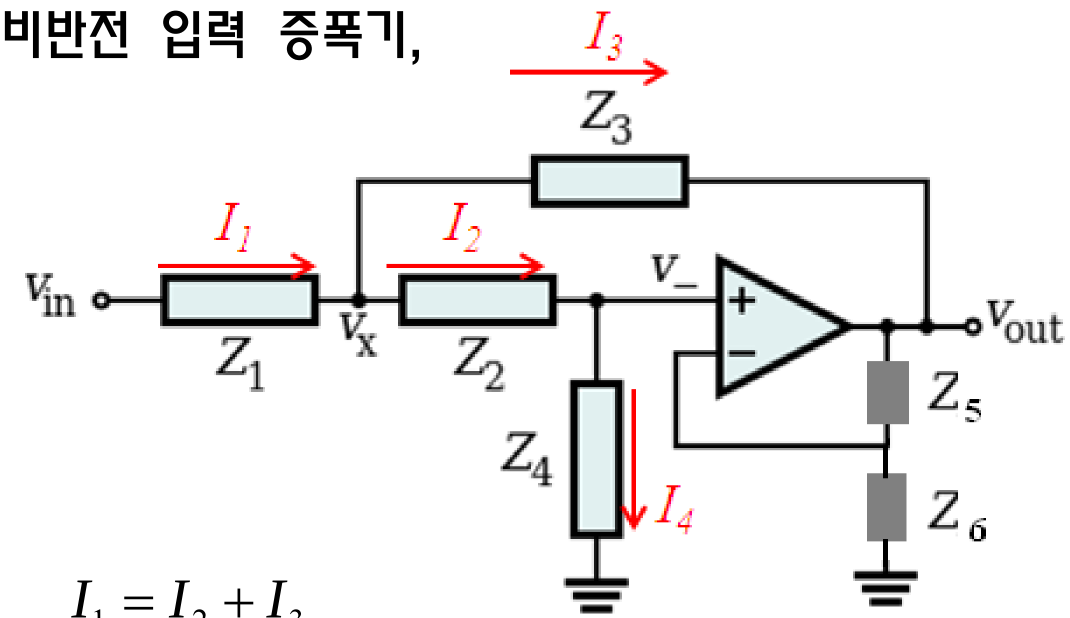
만약 $R_A = R_B = R$, $C_A = C_B = C$

$$f_c = \frac{1}{2\pi RC}$$

For a Butterworth 2nd-order response, DF = 1.414; therefore, $R_1/R_2 = 0.586$.

Sallen-Key 일반 필터 회로

- 비반전 입력 증폭기,



$$I_+ \approx I_- \approx 0$$

$$V_+ \approx V_- = \frac{Z_6}{Z_5 + Z_6} V_{out} = BV_{out}$$

$$I_1 = I_2 + I_3$$

$$\frac{V_{in} - V_x}{Z_1} = \frac{V_x - V_-}{Z_2} + \frac{V_x - V_{out}}{Z_3} \qquad \text{-(1)}$$

$$I_2 = I_4$$

$$\frac{V_x - V_-}{Z_2} = \frac{V_-}{Z_4} \longrightarrow V_x = \left(1 + \frac{Z_2}{Z_4}\right) V_- = \left(1 + \frac{Z_2}{Z_4}\right) BV_{out} \qquad \text{-(2)}$$

Sallen-Key 일반 필터 회로

Yun SeopYu

- 비반전 입력 증폭기,
  - 식 (2)를 식 (1)에 대입하여 전개하면,

$$\frac{V_{in} - \left(1 + \frac{Z_2}{Z_4}\right) BV_{out}}{Z_1} = \frac{\left(1 + \frac{Z_2}{Z_4}\right) BV_{out} - BV_{out}}{Z_2} + \frac{\left(1 + \frac{Z_2}{Z_4}\right) BV_{out} - V_{out}}{Z_3}$$

$$V_{in} - \left(1 + \frac{Z_2}{Z_4}\right) BV_{out} = \left(\frac{Z_1}{Z_4} + \frac{Z_1}{Z_3} + \frac{Z_1}{Z_3}\frac{Z_2}{Z_4} - \frac{1}{B}\frac{Z_1}{Z_3}\right) BV_{out}$$

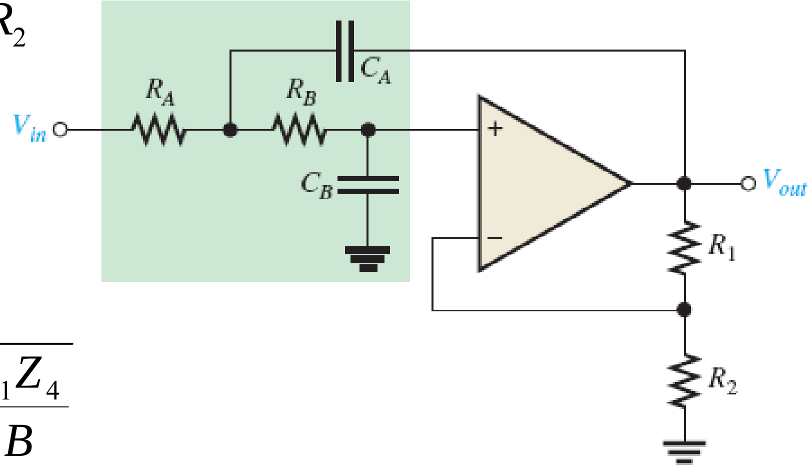
$$A_v = \frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{\left(Z_3 Z_4 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_2 + Z_3 Z_2\right) B - Z_1 Z_4}$$

Sallen-Key 저역 통과 필터

Yun SeopYu

$$Z_1 = R_A, Z_2 = R_B, Z_3 = \frac{1}{sC_A}, Z_4 = \frac{1}{sC_B}, Z_5 = R_1, Z_6 = R_2$$

Two-pole low-pass circuit



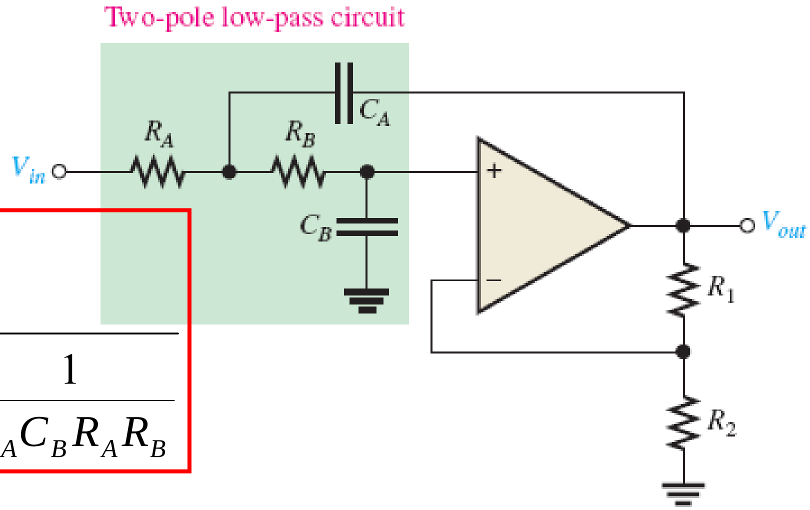
$$A_v = \frac{V_{out}}{V_{in}} = \frac{\dfrac{Z_3 Z_4}{B}}{Z_3 Z_4 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_2 + Z_3 Z_2 - \dfrac{Z_1 Z_4}{B}}$$

$$= \frac{\dfrac{1}{sC_A}\dfrac{1}{sC_B}\dfrac{1}{B}}{\dfrac{1}{sC_A}\dfrac{1}{sC_B} + \dfrac{R_A}{sC_A} + \dfrac{R_A}{sC_B} + R_A R_B + \dfrac{R_B}{sC_A} - \dfrac{R_A}{sC_B}\dfrac{1}{B}}$$

$$= \frac{\dfrac{1}{C_A C_B R_A R_B}\dfrac{1}{B}}{s^2 + s\left[\dfrac{R_A + R_B}{C_A R_A R_B} + \dfrac{1}{C_B R_B}\left(1 - \dfrac{1}{B}\right)\right]\dfrac{(R_1 + R_2)}{C_1 R_1 R_2} + \dfrac{1}{C_A C_B R_A R_B}}$$

Sallen-Key 저역 통과 필터

Yun SeopYu



$$Z_1 = R_A, Z_2 = R_B, Z_3 = \frac{1}{sC_A}, Z_4 = \frac{1}{sC_B}, Z_5 = R_1, Z_6 = R_2$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{\dfrac{1}{C_A C_B R_A R_B}\dfrac{1}{B}}{s^2 + s\left[\dfrac{R_A + R_B}{C_A R_A R_B} + \dfrac{1}{C_B R_B}\left(1 - \dfrac{1}{B}\right)\right]\dfrac{(R_1 + R_2)}{C_1 R_1 R_2} + \dfrac{1}{C_A C_B R_A R_B}}$$

$A_v \longrightarrow$

- 2차 저역통과 필터 전달함수

$$H(s) = \frac{A_0 \omega_0^2}{s^2 + \dfrac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0^2 = (2\pi f_0)^2 = \frac{1}{C_A C_B R_A R_B}$$

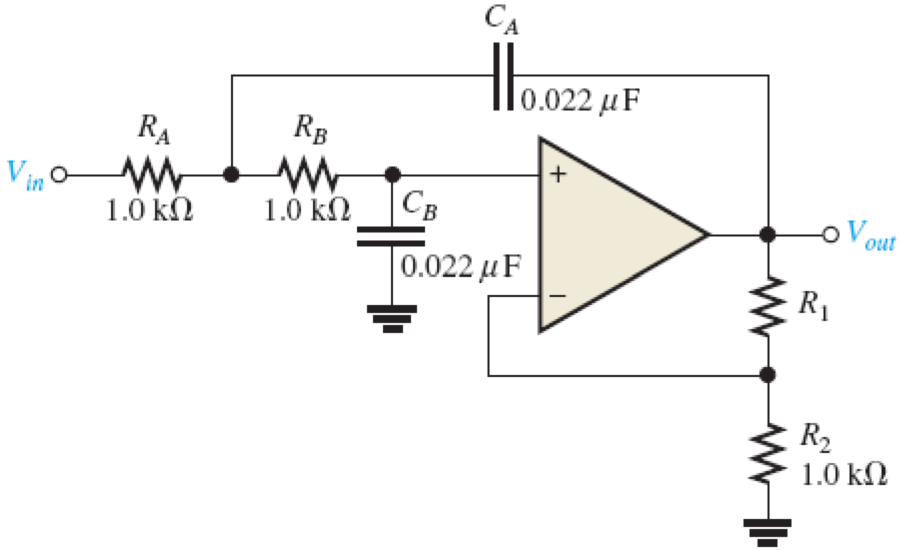
$$-> f_0 = \frac{1}{2\pi\sqrt{C_A C_B R_A R_B}}$$

$$A_0 = 1/B = 1 + R_1/R_2$$

$$\frac{\omega_0}{Q} = \frac{R_A + R_B}{C_A R_A R_B} - \frac{1}{C_B R_B}\left(\frac{R_2}{R_1}\right) --> Q = \omega_0 \frac{1}{\dfrac{R_A + R_B}{C_A R_A R_B} - \dfrac{1}{C_B R_B}\left(\dfrac{R_2}{R_1}\right)} --> Q = \frac{1}{\dfrac{(R_A + R_B)C_B}{C_A R_A R_B} - C_A R_A\left(\dfrac{R_2}{R_1}\right)}$$

예제 15-3

- 저역 통과 필터 → 차단주파수 f_c?, R_1 (버터워스 응답)?
  - $R_A = R_B = R_2 = 1\ k\Omega$, $C_A = C_B = 0.022\ \mu F$



$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}} = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\text{k})(0.022\mu)} = 7.23\text{kHz}$$

2차 버터워스 → DF = 1.414

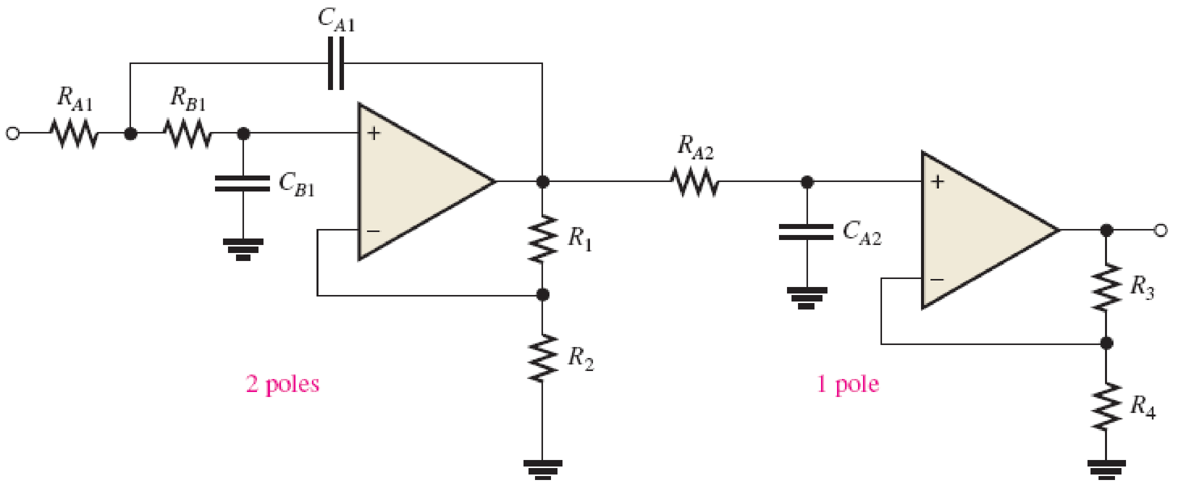
→ $R_1/R_2 = 2 - DF = 2 - 1.414 = 0.586$

→ $R_1 = 0.586R_2 = (0.586)(1k) = 586\ \Omega$

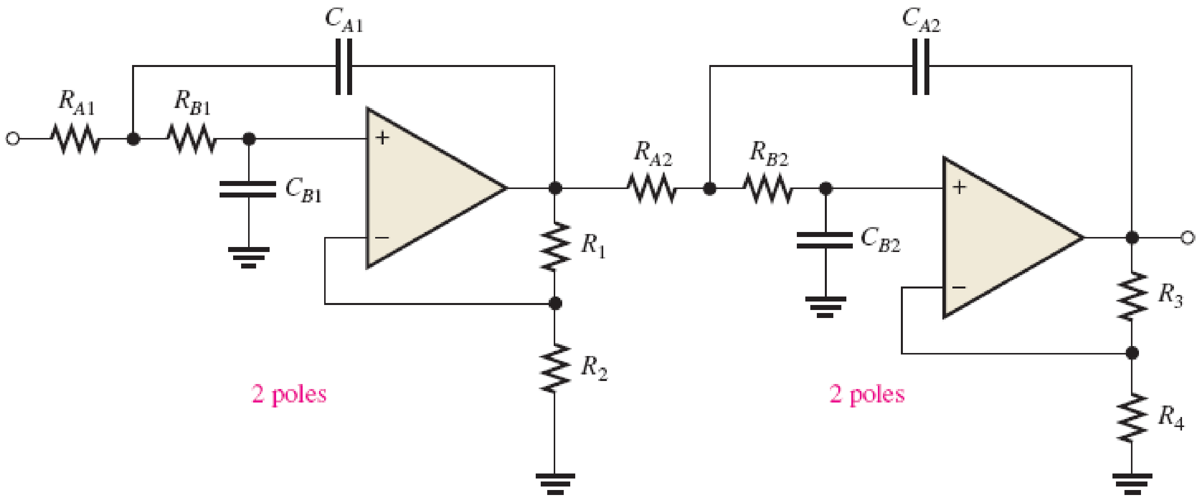
능동 저역 통과 필터

Cascaded Low-Pass Filters

- 높은 기울기를 얻기 위해서 저역통과 필터를 종속 연결함



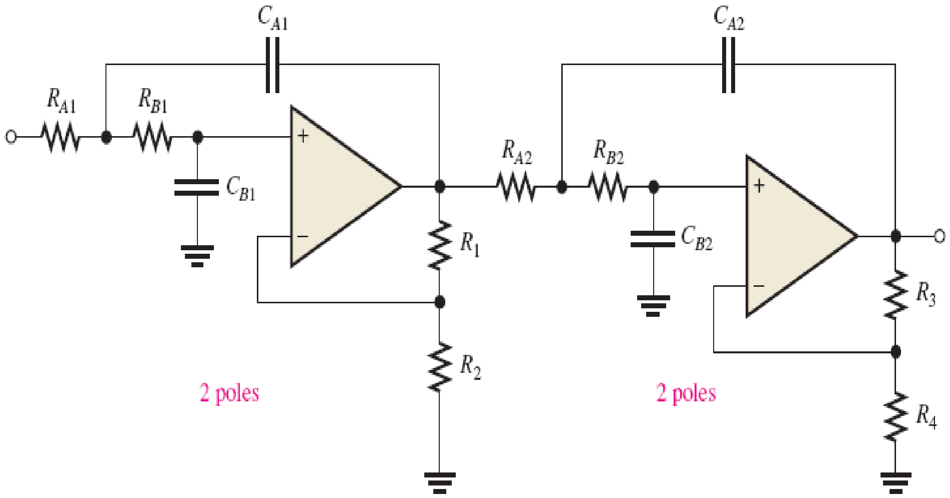
3극: -60 dB/dec



4극: -80 dB/dec

예제 15-4

- 4차 저역통과 필터 → C ?, R_1 = R_3 (버터워스 응답)?
  - $R_{A1} = R_{B1} = R_{A2} = R_{B2} = R_2 = R_4 = 1.8\ k\Omega$
  - $C_{A1} = C_{B1} = C_{A2} = C_{B2} = C$
  - f_c = 2680 Hz 라 하자



$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}} = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.8\text{k})C} = 2.68\text{kHz}$$

$$\longrightarrow \quad C = \frac{1}{2\pi(1.8\text{k})(2.68\text{k})} = 0.033\mu\text{F}$$

4차 버터워스:

- 첫째단: DF = 1.848

→ $R_1/R_2 = 2 - DF$
$= 2 - 1.848 = 0.152$

→ $R_1 = 0.152R_2$
$= (0.152)(1.8\text{k}) = 274\Omega$

- 둘째단: DF = 0.765

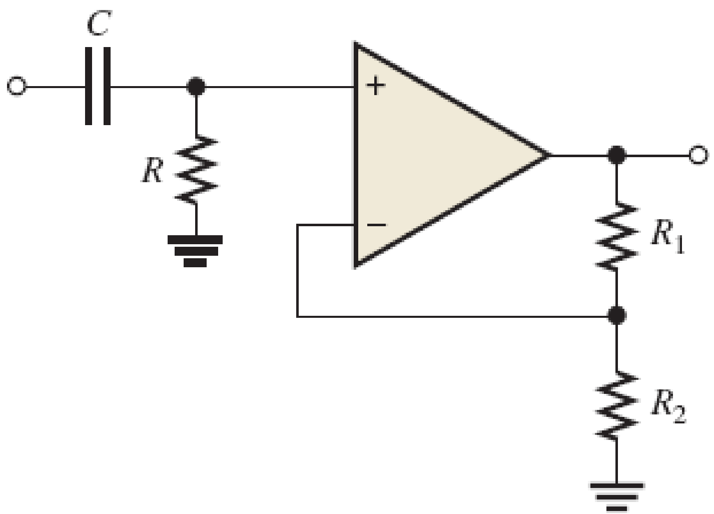
→ $R_3/R_4 = 2 - DF$
$= 2 - 0.765 = 1.235$

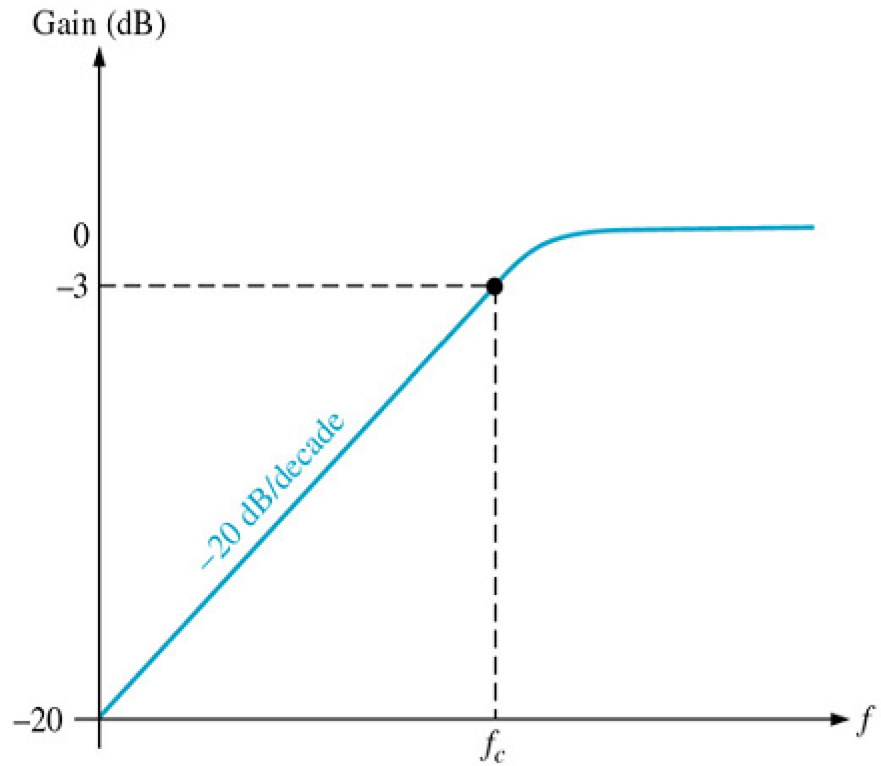
→ $R_3 = 1.235R_4$
$= (1.235)(1.8\text{k})$
$= 2.22\text{k}\Omega$

능동 고역 통과 필터 (Active High-Pass Filter)

Yun SeopYu

- 1차 (단극) 필터

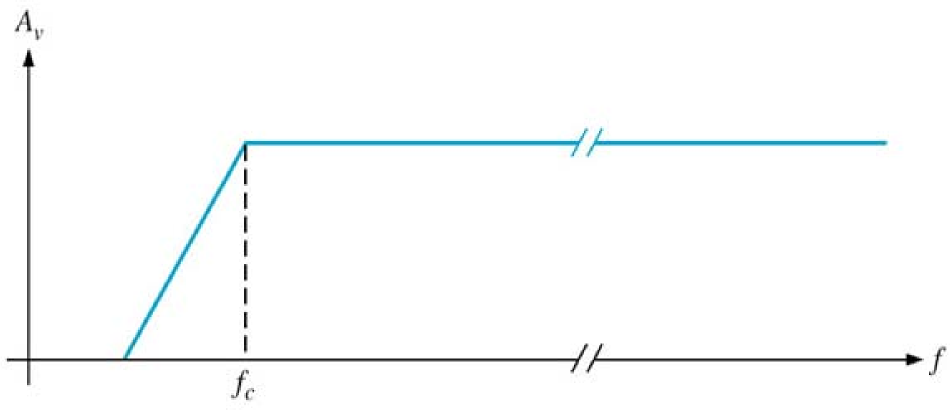




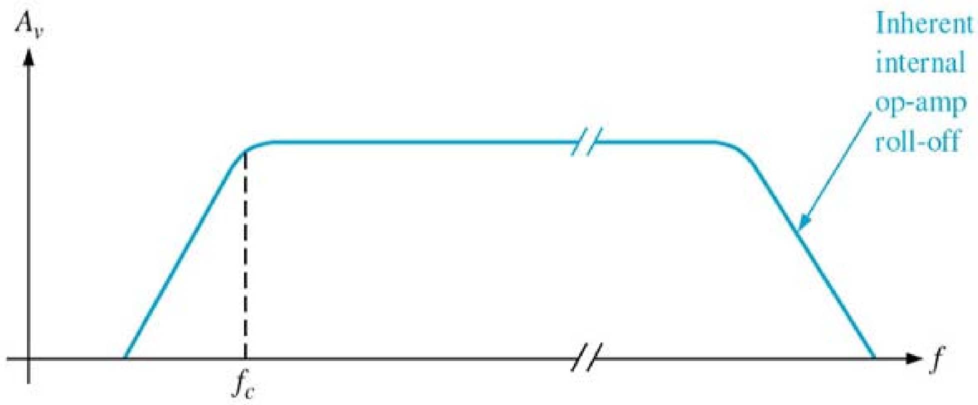
- Roll-off rate, f_c, A_{cl} : LPF와 비슷
- 이상적인 HPF는 f_c 이상의 주파수는 모두 통과
- 그러나, 연산증폭기는 고주파에서 연산증폭기의 내부 RC로 인해 고주파에서 roll-off가 일어남

능동 고역 통과 필터

- 1차 (단극) 필터



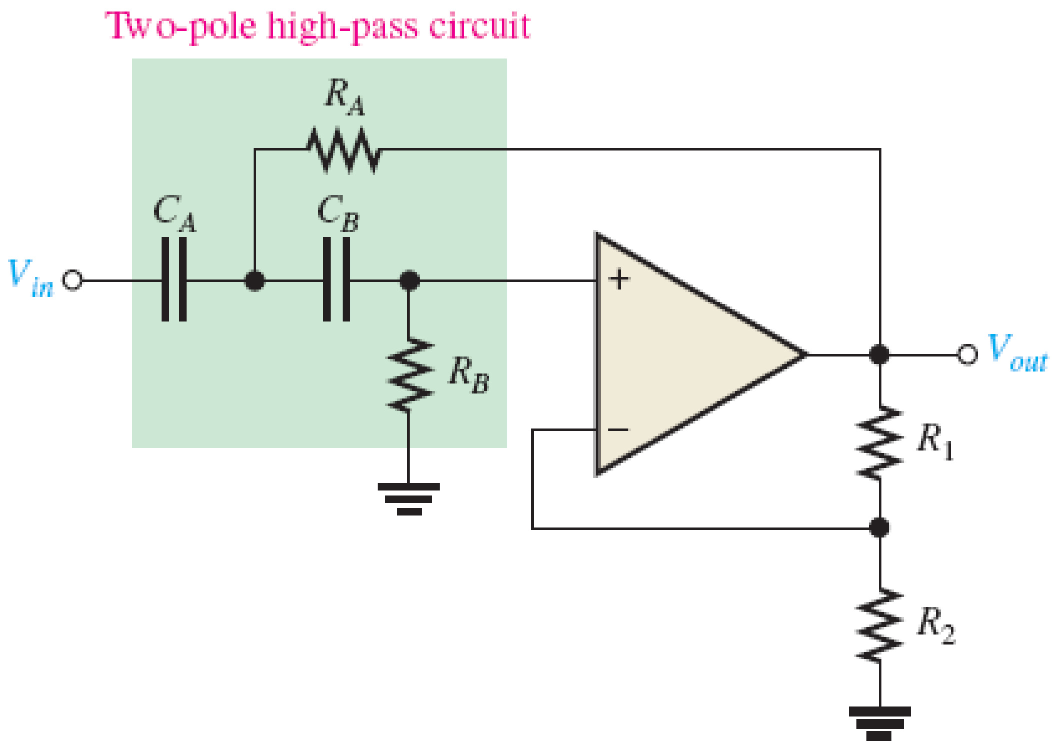
(a) Ideal



(b) Nonideal

능동 고역 통과 필터

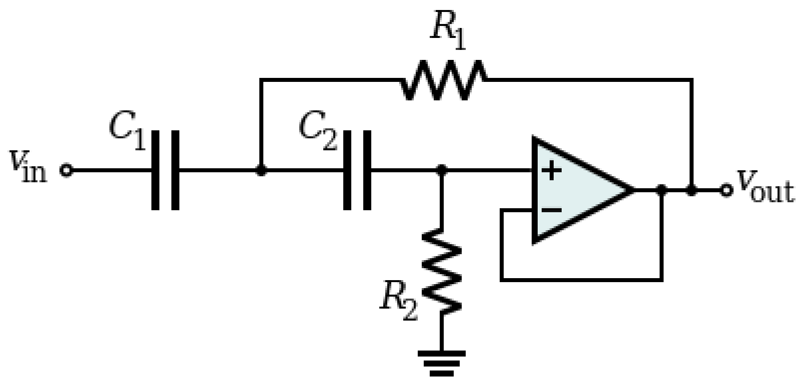
- Sallen-Key 고역통과 필터



→ -40dB/decade의 기울기를 가지는 2개의 고역통과 필터

Sallen-Key 고역 통과 필터

$$Z_1 = \frac{1}{sC_1}, \quad Z_2 = \frac{1}{sC_2}, \quad Z_3 = R_1, \quad Z_4 = R_2$$



$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

$$= \frac{R_1 R_2}{\frac{1}{sC_1}\frac{1}{sC_2} + R_1\left(\frac{1}{sC_1} + \frac{1}{sC_2}\right) + R_1 R_2}$$

$$= \frac{sC_1 sC_2 R_1 R_2}{1 + R_1(sC_1 + sC_2) + sC_1 sC_2 R_1 R_2}$$

$$= \frac{s^2}{s^2 + s\frac{(C_1 + C_2)}{C_1 C_2 R_2} + \frac{1}{C_1 C_2 R_1 R_2}}$$

$A_v \rightarrow$

- 2차 고역통과 필터 전달함수

$$H(s) = \frac{A_0 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$A_0 = 1, \quad \omega_0^2 = (2\pi f_0)^2 = \frac{1}{C_1 C_2 R_1 R_2}$$

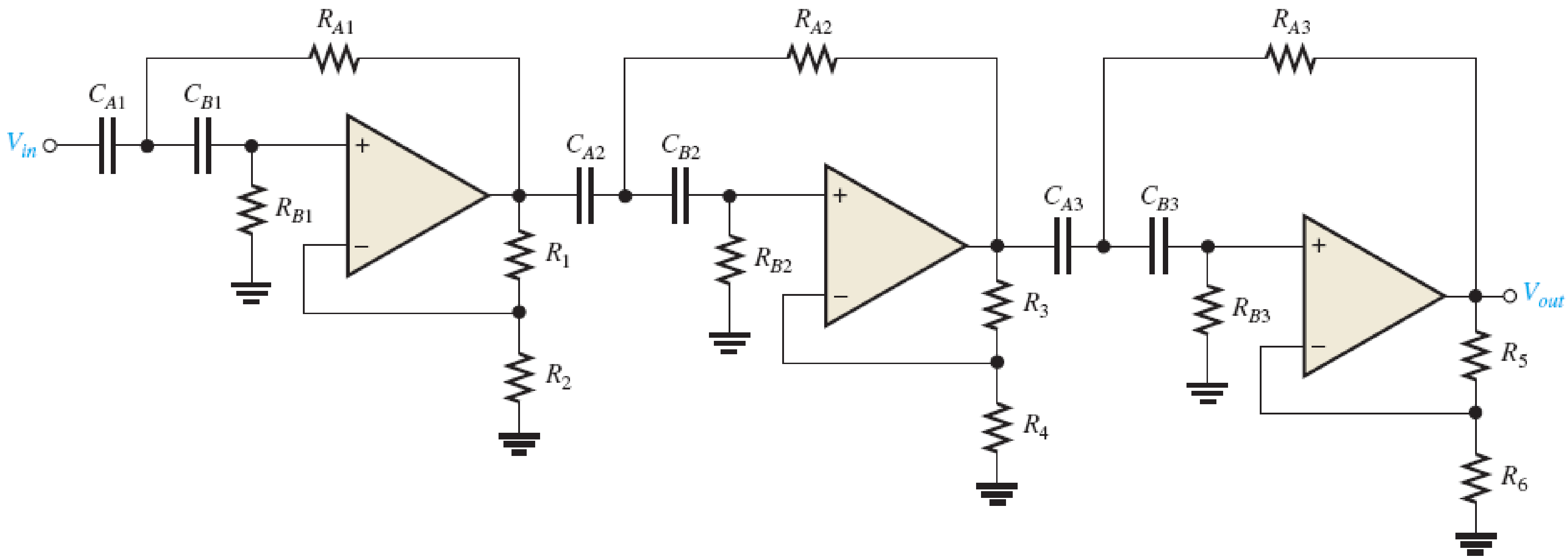
$$-> f_0 = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}}$$

$$Q = \omega_0 \frac{C_1 C_2 R_2}{C_1 + C_2} = \frac{\sqrt{C_1 C_2 R_1 R_2}}{R_1(C_1 + C_2)}$$

$$\frac{\omega_0}{Q} = \frac{C_1 + C_2}{C_1 C_2 R_2} = \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

능동 고역 통과 필터

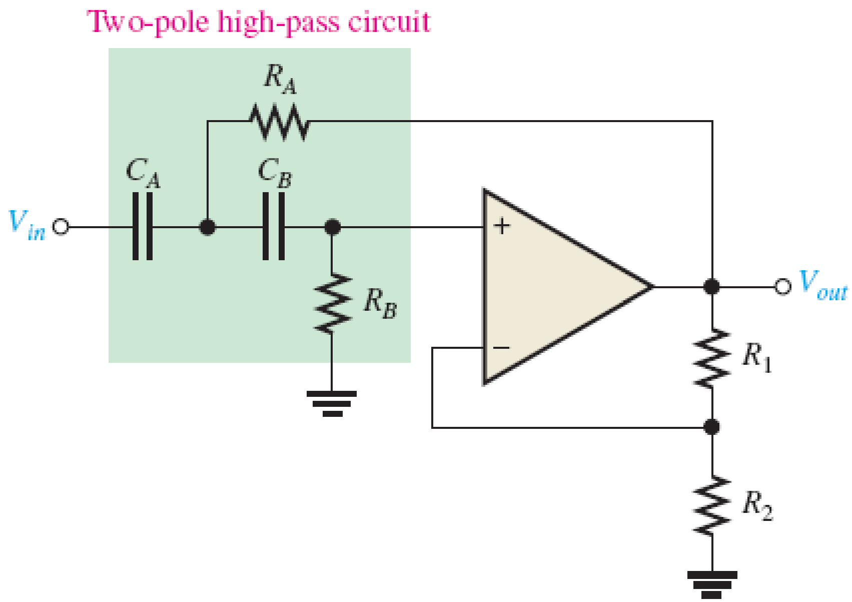
- 종속접속한 고역통과 필터: roll-off 기울기를 증가



6극: -120 dB/dec

예제 15-5

- Shallen-key 고역통과 필터
  - f_c = 10 kHz, 2차 버터워스 응답
  - 가정) R = R_A = R_B = R_2 = 3.3 kΩ
  - C =C_A = C_B라 하자



$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(3.3k)C} = 10\text{kHz}$$

$$\rightarrow C = \frac{1}{2\pi(3.3k)(10k)} = 0.0048\mu\text{F}$$

2차 버터워스 → DF = 1.414

→ R_1/R_2 = 2 – DF = 2- 1.414 = 0.586

→ R_1 = $0.586R_2$ = (0.586)(3.3k) = 1.93 kΩ