Control Engineering

Nyquist 선도

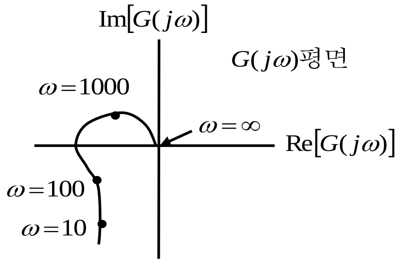
학습목표

- 극좌표선도의 의미와 그리는 법 익히기
- 저주파수와 고주파수 부근에서 점근 거동 이해
- 로그크기 대 위상선도란?

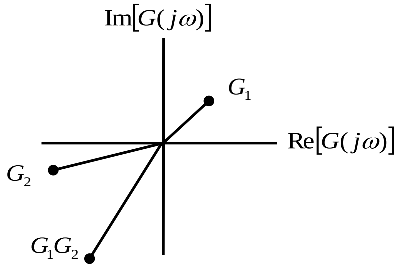
3. Nyquist 선도 (Nyquist plot)

극좌표선도(polar plot)라고도 함.

$$G(s) \rightarrow G(j\omega) = \mathrm{Re}[G(j\omega)] + j\,\mathrm{Im}[G(j\omega)]$$



$$G(s) = G_1(s)G_2(s) \rightarrow G(j\omega) = G_1(j\omega)G_2(j\omega)$$

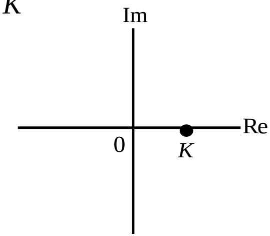


이점 : 한 그림에 주파수응답 특성 표시

불리한 점: 각 factor의 기여도가 나타나기 힘듬

Nyquist 선도

(1) $G(s) = K$

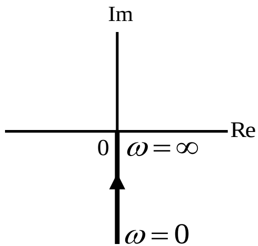


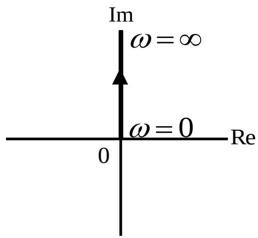
(2) $G(s) = \dfrac{1}{s}$

$G(j\omega) = \dfrac{1}{j\omega}$

$G(s) = s$

$G(j\omega) = j\omega$





(3) $G(s) = \dfrac{1}{1+Ts}$

$$G(j\omega) = \frac{1}{1+T\omega j}$$

$$= \frac{1}{1+\omega^2T^2} + j\frac{(-\omega T)}{1+\omega^2T^2}$$

$$\left.\begin{aligned}\frac{1}{1+\omega^2T^2} &= x \\ \frac{-\omega T}{1+\omega^2T^2} &= y\end{aligned}\right\}\ \omega T\text{를 소거}$$

$$\therefore \left(x-\frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2 \quad \text{원}$$

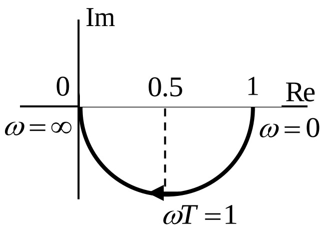
$\omega = 0: \quad G(j\omega) = 1\angle 0°$

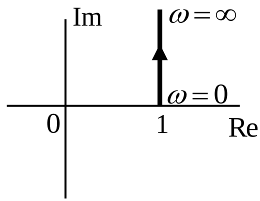
$\omega = 1/T : G(j1/T) = 1/\sqrt{2}\angle -45°$

$\omega = \infty: \quad G(j\infty) = 0\angle -90°$

$G(s) = 1+Ts$

$G(j\omega) = 1+T\omega j$





Nyquist 선도

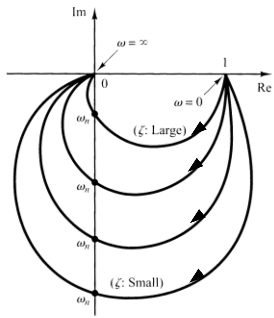
(4) $G(s)=\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, $\qquad G(s)=1+2\zeta\dfrac{s}{\omega_n}+\dfrac{s^2}{\omega_n^2}$

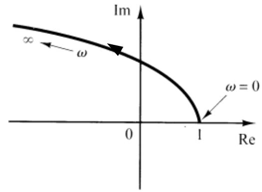
$$=\frac{1}{1+2\zeta\dfrac{s}{\omega_n}+\dfrac{s^2}{\omega_n^2}},\zeta>0$$

$G(j\omega)=\dfrac{1}{1+2\zeta\dfrac{j\omega}{\omega_n}+\dfrac{(j\omega)^2}{\omega_n^2}}$, $\qquad G(j\omega)=1+2\zeta\dfrac{j\omega}{\omega_n}+\dfrac{(j\omega)^2}{\omega_n^2}$

$\omega=0:\quad G(j0)=1\angle 0^\circ,\qquad \omega=0:\ G(j\omega)=1\angle 0^\circ$

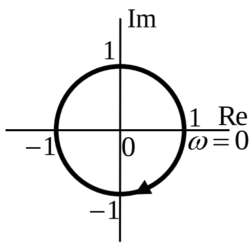
$\omega=\infty:\quad G(j\infty)=0\angle -180^\circ,\qquad \omega=\infty: G(j\omega)=\infty\angle 180^\circ$





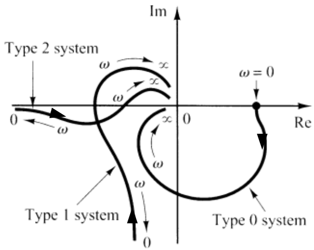
(5) $G(s) = e^{-Ts}$

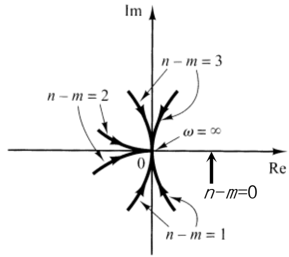
$$G(j\omega) = e^{-T\omega j}$$
$$= 1 \angle -\omega T$$



* Asymptotic behaviors

Low frequency : $\omega = 0$ 부근 High frequency : $\omega = \infty$ 부근





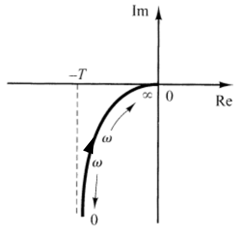
중간 주파수에서 Nyquist 선도의 형태는
numerator dynamics에 의해 영향 받음.

Nyquist 선도

(예) $G(s)=\dfrac{1}{s(1+Ts)}$ 의 Nyquist 선도는?

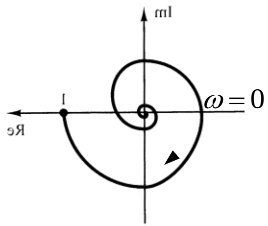
$$\text{Sol}:\ G(j\omega)=\frac{1}{j\omega(1+T\omega j)}=-\frac{T}{1+\omega^2T^2}-j\frac{1}{\omega(1+\omega^2T^2)}$$

As $\omega\to 0,\ \text{Re}[G(j\omega)]\to -T$



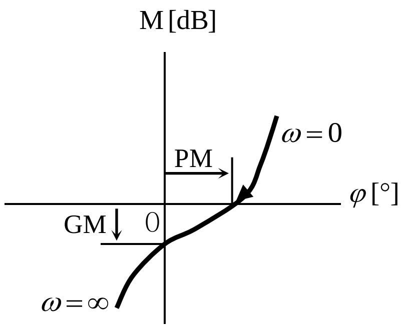
(예) $G(s)=\dfrac{e^{-Ls}}{1+Ts}$ 의 Nyquist 선도는?

$$\text{Sol}:\ G(j\omega)=\frac{e^{-j\omega L}}{1+j\omega L}=\frac{1}{\sqrt{1+\omega^2T^2}}\angle\left(-\omega L-\tan^{-1}\omega T\right)$$



로그크기 대 위상선도

Log-magnitude versus phase plot: Nichols 선도 (Nichols plot)라고도 함. Nichols chart에 그림.



이점: 상대안정도(이득여유 GM, 위상여유 PM)를 쉽게 알 수 있다.

다음 강의

Nyquist 안정도판별법