Economics of Industrial Organization

Lecture 5: Introduction to Game Theory

What is game theory?

- Game theory studies situations where players have strategic interactions; the payoff that one player gets depends not only on the actions taken by that player, but also on the actions of other players.

동시게임
Simultaneous Games

Definition: A *simultaneous* game consists of:

- A set of *players*
- For each player, a set of *actions*
- For each player, a *payoff function* over the set of action profiles.

(As opposed to sequential games, which we consider later.)

내쉬 균형 Nash Equilibrium

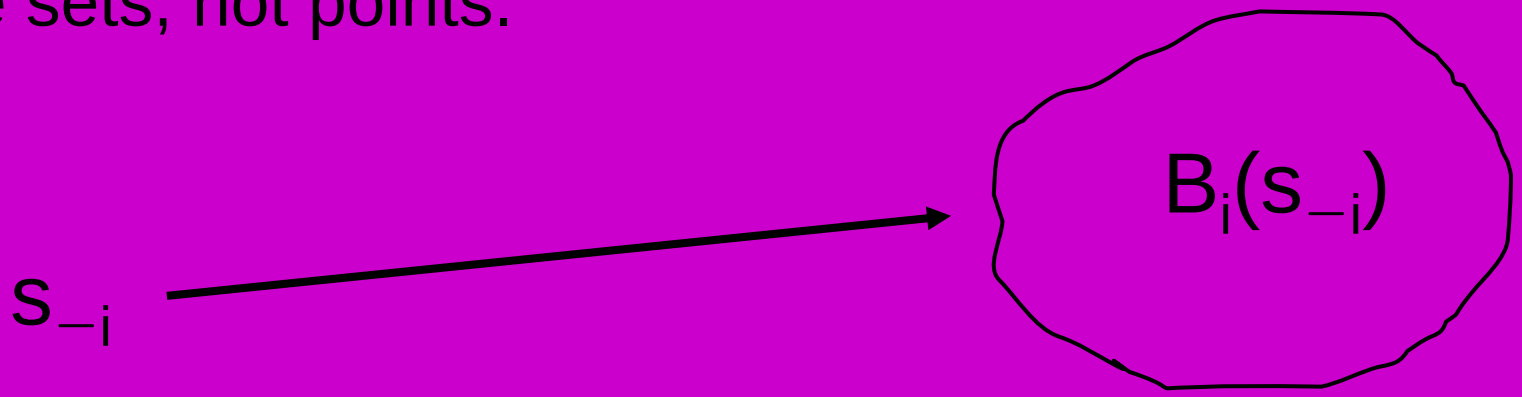
Definition**:** A **strategy** for player *i* describes an action for player *i* to take in every possible circumstance. In a simultaneous move game a (pure) strategy is simply an action.

Definition: A **strategy profile** is a strategy for every player *i*.

Definition: A **Nash equilibrium** is a strategy profile where, given the strategies of every other player, no player can attain a higher payoff by changing their strategy.

최적반응함수
Best Response Functions

- Definition: A player *i*'s **best response** to a given set of strategies for other players is the strategy (or set of strategies) that gives player *i* the highest possible payoff given the strategies of other players.
- In general, best responses are a set-valued function (technically, a correspondence, not a function): its values are sets, not points.



내쉬 균형의 또 다른 정의

Definition: A strategy profile is a **Nash equilibrium** if every player is playing a best response to the strategies of every other player.

죄수의 딜레마 The Prisoner's Dilemma

- Two suspects (Bonnie and Clyde) in a crime, held in separate cells.
- Enough evidence to convict each on minor, but not major offense unless one confesses.
- Each can Remain Silent or can Confess.
- Both remain silent: each convicted of minor offense—1 year in prison.
- One and only one confesses: one who confesses is used as a witness against the other, and goes free; other gets 4 years.
- Both confess: each gets 3 years in prison.

The Prisoner's Dilemma

		Bonnie: RS	Bonnie: C
Clyde	RS	1, 1	4, 0
	C	0, 4	3, 3

- This game models a situation in which there are gains from cooperation, but each player prefers to be a "free rider"

- Unique Nash equilibrium: {C, C}

복점 Duopoly

- Two firms producing same good.
- Each firm can charge high price or low price.
- Both firms charge high price: each gets profit of $1,000
- One firm charges high price, other low: firm charging high price gets no customers and loses $200, while one charging low price makes profit of $1,200
- Both firms charge low price: each earns profit of $600

Duopoly

Firm 2

Firm 1	High	Low
High	1,000, 1,000	-200, 1,200
Low	1,200, -200	600, 600

性대결 Battle of the Sexes

- In the Prisoner's dilemma the players agree that (RS,RS) is a desirable outcome, though each has an incentive to deviate from this outcome.
- Here, the players disagree about the outcome that is desirable.
- Peter and Jane wish to go out together to a concert. The options are U2 or Opera.
- Their main concern is to go out together, but one person prefers U2 and the other person prefers Opera.
- If they go to different concerts then each of them is equally unhappy listening to the music of either side.

Battle of the sexes

		Jane: U2	Jane: Opera
Peter	U2	2, 1	0, 0
	Opera	0, 0	1, 2

동시게임 Simultaneous Games

Common knowledge means that every player knows:

- The list of players
- The actions available to each player
- The payoffs of each player for all possible action profiles
- That each player is a rational payoff-maximizers
- That each player knows that he is rational, and that he knows that everybody else know that he knows they are rational…and so on.

Finding Equilibrium of Game

- *우월전략 Dominant Strategy*: a strategy that is best for a player in a game, regardless of the strategies chosen by the other players.
- A player's action is "strictly dominated" if it is inferior, no matter what the other players do, to some other action.
- Rational players do not play strictly dominated strategies and so, once you determine a strategy is dominated by another, simply remove it from the game.

Iterated Elimination of Strictly Dominated Strategies

		Bonnie: RS	Bonnie: C
Clyde	RS	1, 1	4, 0
	C	0, 4	3, 3

For Clyde, “remain silent” is a dominated strategy. So, “remain silent” should be removed from his strategy space.

Iterated Elimination of Strictly Dominated Strategies

		Bonnie: RS	Bonnie: C
Clyde	RS	1, 1	4, <u>0</u>
	C	<u>0</u>, 4	<u>3</u>, <u>3</u>

Given the symmetry of the game, it is easy to see that “remain silent” is a dominated strategy for Bonnie also. So, “remain silent” should be removed from her strategy space, too.

Iterated Elimination of Strictly Dominated Strategies

Clyde \ Bonnie	RS	C
RS	1, 1	4, 0
C	0, 4	3, 3

Given the symmetry of the game, it is easy to see that “remain silent” is a dominated strategy for Bonnie also. So, “remain silent” should be removed from her strategy space, too.

Iterated Elimination of Strictly Dominated Strategies

		Player 2		
		Left	**Middle**	**Right**
Player 1	**Up**	1,0	1,2	0,1
	Down	0,3	0,1	2,0

Iterated Elimination of Strictly Dominated Strategies

		Player 2		
		Left	**Middle**	**Right**
Player 1	**Up**	1,0	1,2	0,1
	Down	0,3	0,1	2,0

Iterated Elimination of Strictly Dominated Strategies

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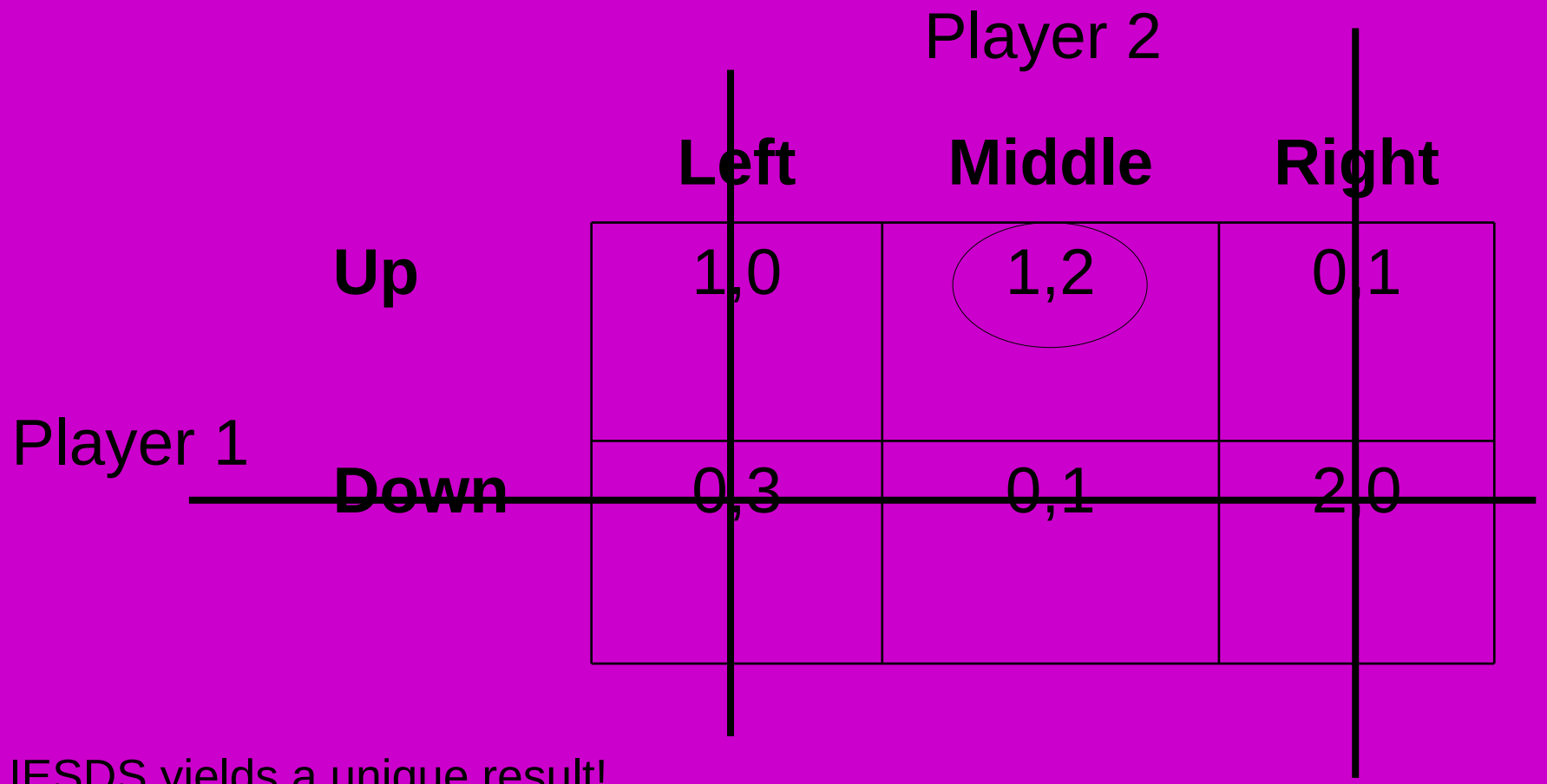

IESDS yields a unique result!

Iterated Elimination of Strictly Dominated Strategies

		Player 2		
		Left	**Middle**	**Right**
Player 1	**Top**	0,4	4,0	5,3
	Middle	4,0	0,4	5,3
	Bottom	3,5	3,5	6,6

In many cases, iterated elimination of strictly dominated strategies may not lead to a unique result. Here, we cannot eliminate any strategies.

Weakly Dominated Strategies

A player's action "weakly dominates" another action if the first action is at least as good as the second action, no matter what the other players do, and is better than the second action for some actions of the other players. (Also known as Pareto dominates)

Weakly dominated strategies

- We cannot always do iterated elimination if weakly dominated strategies in the same way that we could for strictly dominated strategies
- Sometimes the set of actions that remains depends on the order in which we eliminated strategies. Consider eliminating L, T vs R, B.

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1	L	C	R
T	1,1	1,1	0,0
B	0,0	1,2	1,2

Nash equilibrium and elimination

- All Nash equilibria will survive iterated elimination of strictly dominated strategies.
  Thus, if multiple Nash equilibria are present, then iterated elimination of strictly dominated strategies will not leave us with a unique strategy for all players.
- *This is not true of iterated elimination of weakly dominated strategies: we can sometimes eliminate Nash equilibria by eliminating weakly dominated strategies.*

Synergistic Relationship

- Two individuals
- Each decides how much effort to devote to relationship
- Amount of effort is nonnegative real number
- If both individuals devote more effort to the relationship, then they are both better off; for any given effort of individual j, the return to individual i's effort first increases, then decreases.
- Specifically, payoff of i: $a_i(c + a_j - a_i)$, where $c > 0$ is a constant.

Synergistic Relationship

Payoff to player i: $a_i(c+a_j-a_i)$.

First Order Condition: $c + a_j - 2a_i = 0$.

Second Order Condition: $-2 < 0$, Max

Solve FOC: $a_i = (c+a_j)/2 = b_i(a_j)$

Symmetry implies: $a_j = (c+a_i)/2 = b_j(a_i)$

Synergistic Relationship

Symmetry of quadratic payoff functions implies that the best response of each individual *i* to a_j is

$$b_i(a_j) = 1/2\ (c + a_j)$$

Nash equilibria: pairs (a^*_1, a^*_2) that solve the two equations

$$a_1 = 1/2\ (c + a_2)$$

$$a_2 = 1/2\ (c + a_1)$$

Unique solution, (c, c)

Hence game has a unique Nash equilibrium

$$(a^*_1, a^*_2) = (c, c)$$