Digital Communications
KEEE346_02
Extra Note
Lecture Note 13

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Maximum-Likelihood Sequence Detector (MLSD)

- When the signal has no memory (ex., no ISI), the symbol-by-symbol detector is optimum in the sense of minimizing the probability of a symbol error.
- On the other hand, when the transmitted signal has memory (that is, there exists ISI) such that the signals transmitted in successive symbol intervals are interdependent, the optimum detector is a detector that bases its decisions on observation of a sequence of received signals over successive signal intervals.
- In this class, we describe a maximum-likelihood sequence detection algorithm that searches for the minimum Euclidean distance path through the trellis that characterizes the memory in the transmitted signal.
- To develop the MLSD algorithm, let us consider, as an example, the binary PAM.
  - Hence, there are two possible transmitted signals corresponding to the signal points

$$s_1 = -s_2 = \sqrt{E_b}$$

  - The output of the matched filter or correlation demodulator for binary PAM in the kth signal interval may be expressed as

$$r_k = \pm\sqrt{E_b} + n_k$$

    - where n_k is a zero-mean Gaussian random variable with variance $\sigma_n^2 = N_0/2$

- The conditional PDFs for the two possible transmitted signals are

$$
\begin{aligned}
f(r_k|s_1) &= \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r_k-\sqrt{E_b})^2}{2\sigma_n^2}\right] \\
f(r_k|s_2) &= \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r_k+\sqrt{E_b})^2}{2\sigma_n^2}\right]
\end{aligned}
$$

- Now we suppose we observe the sequence of matched-filter outputs $r_1,\ r_2,\ \cdots, r_K$.
  - Since the channel noise is assumed to be white and Gaussian, and $f(t-iT)$ and $f(t-jT)$ are orthogonal for $i \neq j$, it follows that $E(n_k n_j)=0,\ \ k\neq j$
    - Hence the noise sequence $n_1, n_2, \ldots, n_K$ is also white.
  - Consequently, for any given transmitted sequence $\mathbf{s}^{(m)}$, the joint PDF of $r_1, r_2, \cdots, r_K$ may be expressed as a product of K marginal PDFs, i.e.,

$$
\begin{aligned}
f(r_1, r_2, \ldots, r_K|\mathbf{s}^{(m)}) &= \prod_{k=1}^{K} f(r_k|s_k^{(m)}) \\
&= \prod_{k=1}^{K} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r_k-s_k^{(m)})^2}{2\sigma_n^2}\right] \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma_n}\right)^K \exp\left[-\sum_{k=1}^{K}\frac{(r_k-s_k^{(m)})^2}{2\sigma_n^2}\right]
\end{aligned}
$$

- where either $s_k = \sqrt{E_b}$ or $s_k = -\sqrt{E_b}$.
- Then, given the received sequence $r_1, r_2, \cdots, r_K$ at the output of the matched filter or correlation demodulator, the detector determines the sequence

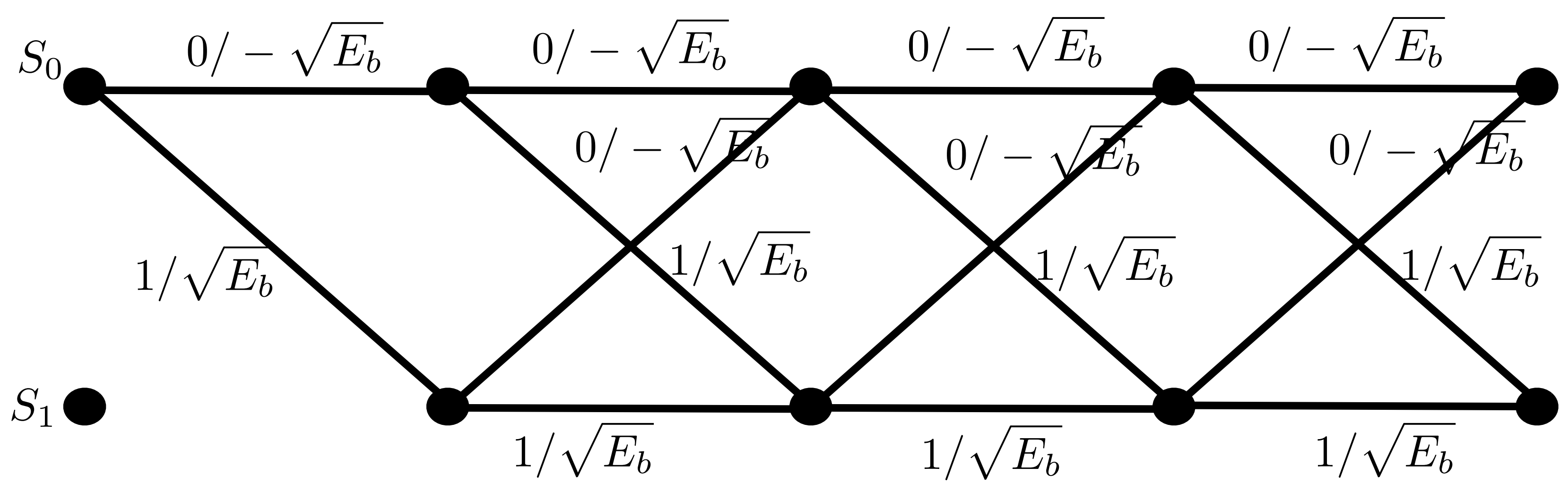
$$\mathbf{s}^{(m)} = \{s_1^{(m)}, s_2^{(m)}, \ldots, s_K^{(m)}\}$$

  - which maximizes the conditional PDFs $f(r_1, r_2, \ldots, r_K | \mathbf{s}^{(m)})$.
  - Such a detector is called the maximum-likelihood sequence detector (MLSD).
- By taking the natural logarithm of conditional PDFs and neglecting terms that are independent of $(r_1, r_2, \ldots, r_K)$, we find that an equivalent ML sequence detector selects the sequence $\mathbf{s}^{(m)}$ that minimizes the *Euclidean distance metric*

$$D(\mathbf{r}, \mathbf{s}^{(m)}) = \sum_{k=1}^{K} (r_k - s_k^{(m)})^2$$

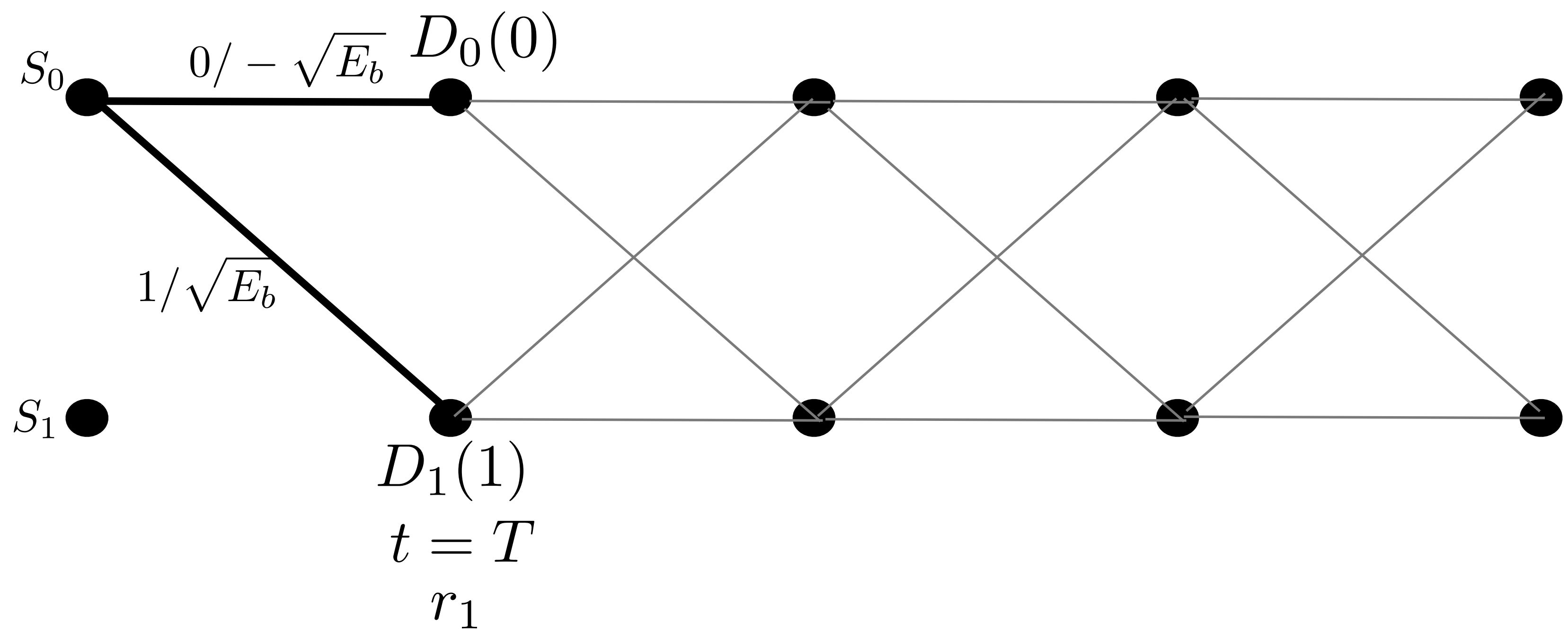
- In searching through the trellis for the sequence that minimizes the Euclidean distance it may appear that we must compute the distance $D(\mathbf{r}, \mathbf{s}^{(m)})$ for every possible sequence.
  - For binary PAM example, which employs binary modulation, the total number of sequence is 2^K, where K is the number of outputs obtained from the demodulator.
  -

- However, this is not the case. We may reduce the number of sequences in the trellis search by using the *Viterbi algorithm* to eliminate sequences as new data is received from the demodulator.
- The Viterbi algorithm is a sequential trellis search algorithm performing MLSD.
  - The Viterbi algorithm will be described in detail for decoding the convolutional coding in the class of Information and Channel coding theory.
  - We describe it here in the context of binary PAM.
- Viterbi algorithm
  - We assume that the search process begins initially at state S_0 .



  - Viterbi algorithm consists of 'Add', 'Compare' and 'Select' (ACS) procedures.

- Add
  - At every transition we calculate the Euclidean distance. For example, at time t=T, we receive $r_1 = s_1^{(m)} + n_1$ and we can calculate the two Euclidean distance metrics.

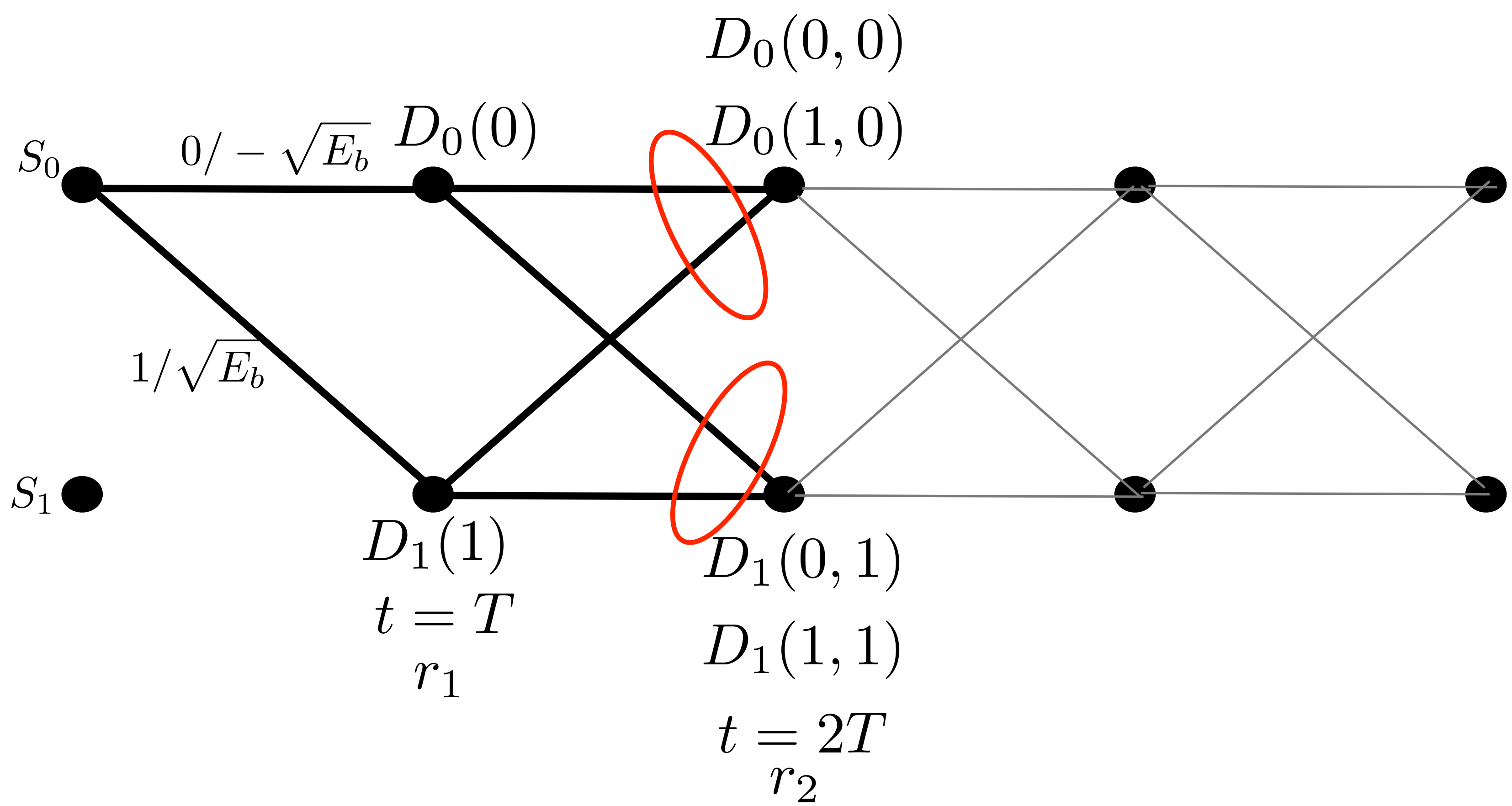


  - At t=2T, for the two paths entering node S_0, we compute the two Euclidean distance metrics:

$$
\begin{aligned}
D_0(0,0) &= (r_1 + \sqrt{E_b})^2 + (r_2 + \sqrt{E_b})^2 \\
D_0(1,0) &= (r_1 - \sqrt{E_b})^2 + (r_2 + \sqrt{E_b})^2
\end{aligned}
$$

  - For the two paths entering node S_1, we have

$$
\begin{aligned}
D_1(0,1) &= (r_1 + \sqrt{E_b})^2 + (r_2 - \sqrt{E_b})^2 \\
D_0(1,1) &= (r_1 - \sqrt{E_b})^2 + (r_2 - \sqrt{E_b})^2
\end{aligned}
$$



- at T=2T, we can calculate the Euclidean distance metrics as follows

$$
\begin{aligned}
D_0(0,0) &= D_0(0) + (r_2 + \sqrt{E_b})^2 \\
D_0(1,0) &= D_1(1) + (r_2 + \sqrt{E_b})^2 \\
D_1(0,1) &= D_0(0) + (r_2 - \sqrt{E_b})^2 \\
D_1(1,1) &= D_1(1) + (r_2 - \sqrt{E_b})^2
\end{aligned}
$$

- Generally, we can calculate two metrics at every node by adding the previous accumulated metric values to the distance between the current received symbol and the possible symbol.

- Compare and Select
  - At every node, there are two entering paths.
  - The metric values of those two entering paths are compared.
    - For example, at t=2T, we compare $D_0(0,0)$ and $D_0(1,0)$ at node S_0 and select only one path which has the smaller value of the Euclidean distance metric.

- This process is continued as each new signal sample is received from the demodulator. Thus, the Viterbi algorithm computes two metrics for the two signal paths entering a node at each stage of the trellis search and eliminates one of the two paths at each node.
  - The two survivor paths are then extended forward to the next state.
  - Therefore, the number of paths searched in the trellis is reduced by a factor of 2 at each stage.
- From the description of the Viterbi algorithm given above, it is unclear as to how decisions are made on the individual detected information symbols given the surviving sequences.
  - Let us assume that the channel has a memory of 3 bits.
  - If we have advanced to some stage, say K, where K>>L in the trellis, we compare the surviving sequences, it is known that 5L bits calculations for surviving paths and making decisions are quite approaching to the optimal case.

MLSD over ISI Channel

- The output sample value at the output of the demodulator over ISI channel can be written as

$$y_n = I_n + \sum_{k=1}^{L} I_{n-k} x_k + \nu_k$$

  - where L is the length of the memory.

- Then MLSD minimizes the following Euclidean distance metric:

$$\sum_{n=1}^{K} \left[y_n - (I_n + \sum_{k=1}^{L} I_{n-k} x_k) \right]^2$$

  - We can calculate the path metric using the Viterbi algorithm.

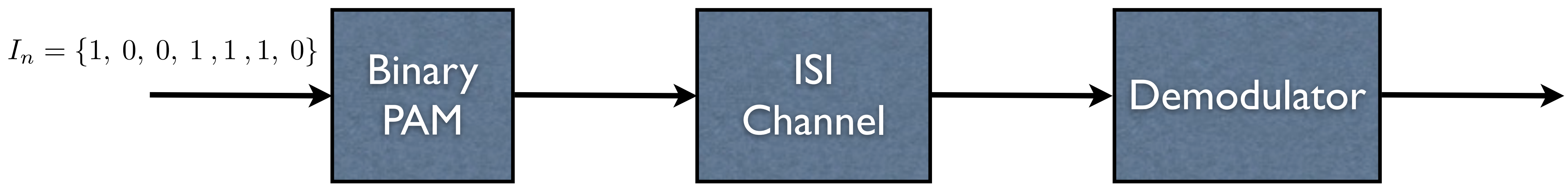
Example

- Received signal sample at the output of the demodulator over three bit of memory channels can be expressed as

$$y_n = I_n + \alpha I_{n-1} + \nu_n$$

where $I_n \in \{\sqrt{E_b}, -\sqrt{E_b}\}$.

Assume that $E_b = 1, \quad \alpha = \exp(-1) = 0.3679$. The binary PAM modulates the binary bit 0 into $-\sqrt{E_b}$ and the binary bit into $\sqrt{E_b}$.



If the received signal sample values at the output of the demodulator is given as

$$y = [0.7350,\ -0.6499,\ -1.4331,\ 0.5363,\ 1.0715,\ 1.2830,\ -0.7197]$$

which is the case that the sampled noise is given as

$$\nu = [-0.2650,\ -0.0178,\ -0.0652,\ -0.0958,\ -0.2964,\ -0.0849,\ -0.0875]$$

- Assuming that the first transmit bit is always one, detect the received signal based on the MLSD using the Viterbi algorithm.