

10. Continuity Equation of Minority Carrier

【Key Points】

To learn the minority carrier injection and the continuity equation in the semiconductor.

• In case of injecting the minority carrier (electron)

Suppose the carrier density of thermal equilibrium states before carrier injection is n_{p0} and after carrier injection is n_p .

(Suffix p means that the carrier is in p-type semiconductor, 0 means that the carrier is in the thermal equilibrium states.)

Here the excess minority carrier density is defined as Δn_p .

$$\Delta n_p = n_p - n_{p0}$$

Suppose the condition that the injection carrier density is low,

$$n_{p0} < \Delta n_p < p_{p0}$$

When the minority carriers (electron) are injected in thermal equilibrium states...

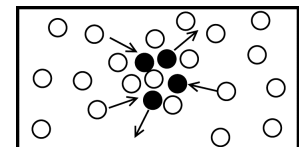
1. Electrons are surrounded by holes due to the Coulomb force.

(Response within dielectric relaxation time)



Electric fields by electron fields disappear in a short time.

(It returns as charge neutrality states)



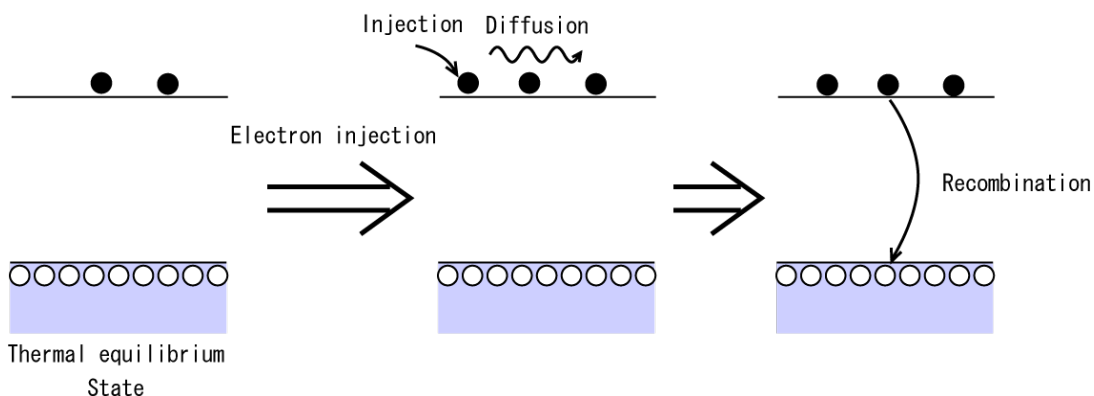
2. Electrons are localized in semiconductor.

→ Electrons diffuse in a direction to the lower density position.

3. Electrons tend to be recombined with hole because of energy gain.

→ Electrons in the conduction band bind together with holes in the valence band. (Recombination)

4. Diffusion and recombination occur to realize the thermal equilibrium states.



• Lifetime of minority carrier

”Lifetime” is the time from the generation to the recombination of the minority carriers.

Ex.) The semiconductor is exposed to light with high energy.

→ Excess minority carriers are generated by the exposure.

Let light stop at the time $t=0$. Excess minority carriers gradually disappear due to the recombination. The rate of minority carrier disappearance is proportional to the difference of carrier density and let the proportional constant be $1/\tau_e$, the equation of expression of this phenomena is,

$$\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_e}$$

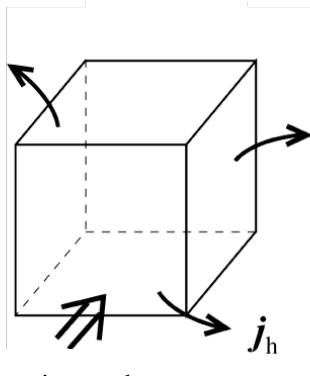
Therefore, the excess minority carrier density is expressed as,

$$\Delta n_p(t) = \Delta n_p(0) \exp\left(-\frac{t}{\tau_e}\right)$$

The excess minority carrier density decreases exponentially with time. τ_e is the lifetime, which express the rate of decreasing the excess minority carrier.

• Continuity equation of minority carrier :

Time dependence of carrier increase and decrease.



The cause of increasing and decreasing the minority carrier in n -type semiconductor

- ① Hole current (j_h)
- ② Recombination
- ③ Generation (by temperature, light...)

Let G is the number of electron-hole pair generated at 1 cm^{-3} per second.

$$\begin{aligned} \frac{\partial p_n}{\partial t} &= (\text{Current}) + (\text{Recombination}) + (\text{Generation}) \\ &= -\frac{1}{q} \text{div} \mathbf{j}_h - \frac{p_n - p_{n0}}{\tau_h} + G \end{aligned} \quad (1)$$

(τ_h : Hole lifetime, p_{n0} : Hole density at thermal equilibrium states)

From here, we consider the one-dimensional states.

$$\rightarrow \text{div} \Rightarrow \frac{\partial}{\partial x}$$

Electrical current consists of the drift and diffusion current. Therefore, whole current is expressed as,

$$j_h = q \left(p_n \mu_h F - D_h \frac{dp_n}{dx} \right) \quad (2)$$

Combine equations (1) and (2), the following equation can be obtained.

$$\frac{\partial p_n}{\partial t} = -\mu_h F \frac{\partial p_n}{\partial x} + D_h \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_h} + G$$

(Continuity equation of minority carrier)

In *p*-type semiconductor, the continuity equation is,

$$\frac{\partial n_p}{\partial t} = \mu_e F \frac{\partial n_p}{\partial x} + D_e \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{p0}}{\tau_e} + G$$

In steady state (carrier density is constant with time), the continuity equations of minority carriers are,

$$-\mu_h F \frac{\partial p_n}{\partial x} + D_h \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_h} + G = 0$$

$$\mu_e F \frac{\partial n_p}{\partial x} + D_e \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{p0}}{\tau_e} + G = 0$$

Actual analysis methods (self-consistent analysis)

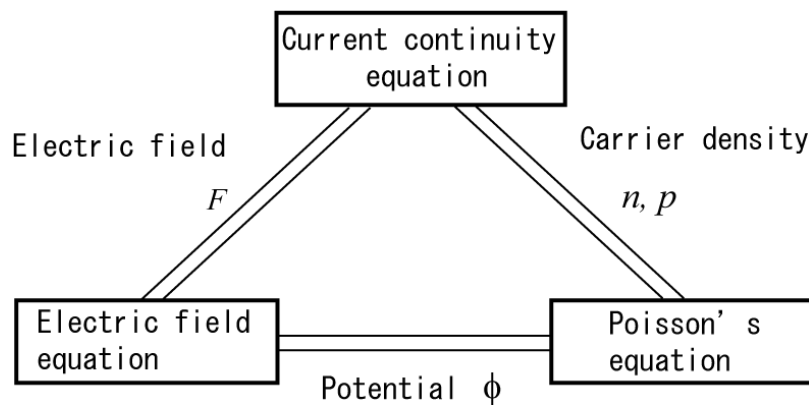
The continuity equation has the term containing electric field F . Electric field F is expressed as

$$F = -\text{grad}\phi \quad (\phi : \text{electrostatic potential})$$

From Poisson's equation, ϕ is expressed as

$$\nabla^2\phi = \text{div}(\text{grad}\phi) = -\frac{q}{\epsilon_r\epsilon_0}(N_D - n + p - N_A)$$

The Poisson's equation has the electron density n and the hole density, and relates with the continuity equation. Therefore, all physical quantity should be consistent in these equations and the analysis must satisfy the assumption. This method is called as "self-consistent analysis"



$$\frac{\partial p}{\partial t} = -\mu_h F \frac{\partial p}{\partial x} + D_h \frac{\partial^2 p}{\partial x^2} - \frac{p - p_0}{\tau_h} + G$$

$$\frac{\partial n}{\partial t} = \mu_e F \frac{\partial n}{\partial x} + D_e \frac{\partial^2 n}{\partial x^2} - \frac{n - n_0}{\tau_e} + G$$

$$F = -\text{grad}\phi$$

$$\nabla^2\phi = \text{div}(\text{grad}\phi) = -\frac{q}{\epsilon_r\epsilon_0}(N_D - n + p - N_A)$$